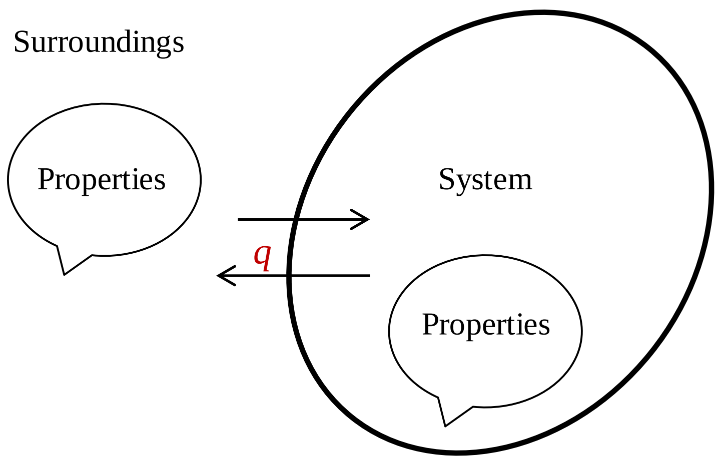
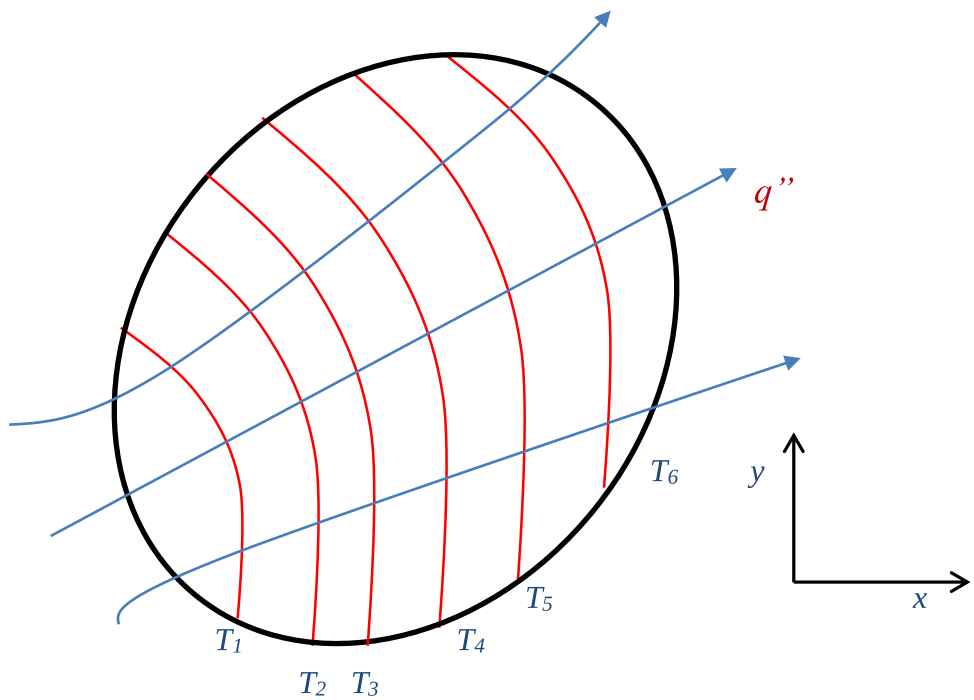
Heat transfer mechanisms and modes

Heat is a sort of energy interaction between a system and its surroundings.

It is originated from temperature difference between the system and the surroundings.



Heat is a transfer quantity, is not a system property.



If $T_1 > T_2 > T_3 > T_4 > T_5 > T_6$, heat flow in unit area (heat flux) perpendicular to the isotherms in a proportional manner as

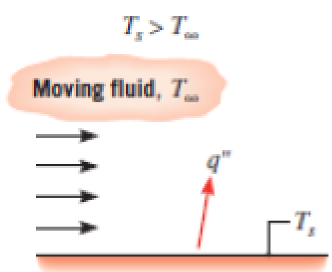
$$q'' \propto \frac{dT}{dx}$$

Here dT/dx is a property gradient and q'' a transfer quantity, the above proportionality indicates relation between this quantity and the gradient. The proportionality constant is a physical property of the matter that the transfer taking place. Then the proportionality becomes anequality by

$$q'' = -k\frac{dT}{dx}$$

That is well known ***Fourier's Law of Heat Conduction***

Another mechanism comes from our experience is the heat transfer between a surface and a moving fluid as indicated the illustration below called convection.



In this case the proportionality exist between temperature difference $(T_s - T_\infty)$ and the heat flux q'' as

$$q'' \propto (T_s - T_\infty)$$

Then the convection heat transfer coefficient h relates the heat and the temperature difference as

$$q'' = h(T_s - T_\infty)$$

is the convection equation known as ***Newton's Law of Cooling***.

- ✓ The third heat transfer mode is heat radiation which is quite different from the first two mechanisms.

- ✓ The conduction takes place at place no matter the substance is solid, liquid or gas.

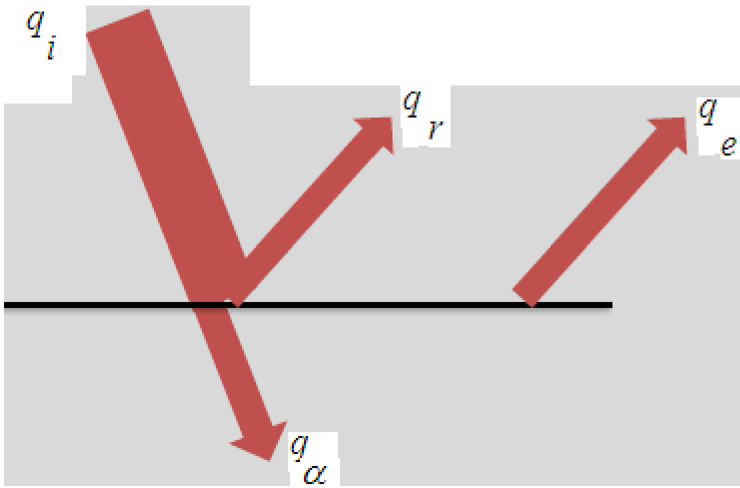
- ✓ The convection can be only occurs in a fluid (liquids and gasses) because particles (atoms, molecules, etc.) must be replaced from one point to another in order to convection transfer sustained.

- Radiation is phenomena that is basically different from the conduction and the convection in two ways:
  - It can take place also through non-existing matter (vacuum, space etc.)
  - Without requiring a continuing function of temperature.

  A clear example of the mechanism is the heat gain of the earth from the sun (or heat loss of the earth to the space)

- Each surface at a finite temperature emits $\varepsilon\sigma T^4$ amount of thermal energy by electromagnetic waves.
- This fact known as ***Stefan-Boltzman Law of Thermal Radiation***.

Here ε is the surface emissivity and takes the value as $0 < \varepsilon < 1$. σ is the Stefan-Boltzman constant and has the constant value of $\sigma = 5.67 \times 10^{-8}$ W/m^2K^4



q_i : Incoming radiation

q_r : Reflection

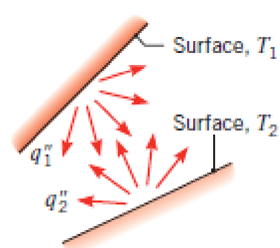
q_e : Emission

q_α : Absorption

Absorptivity of a surface α also takes the value of $0 < \alpha < 1$.

If $\alpha = \varepsilon$ is the case it is called Gray Surface.

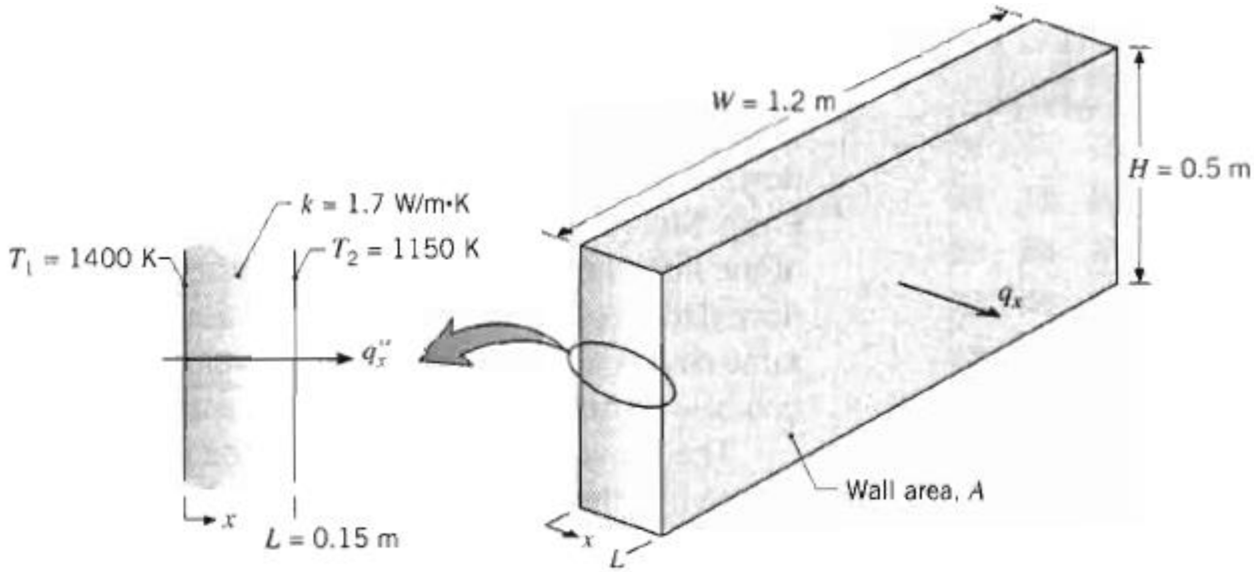
Then net radiant heat exchange between the surfaces indicated in the figure is



$$q'' = \varepsilon\sigma(T_1^4 - T_2^4)$$

Example problem:

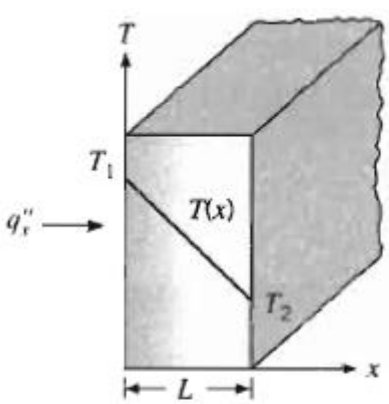
The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m · K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m by 1.2 m on a side?



For the steady state condition temperature profile is linear,

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

Then the Fourier law becomes



One-dimensional heat transfer by conduction (diffusion of energy).

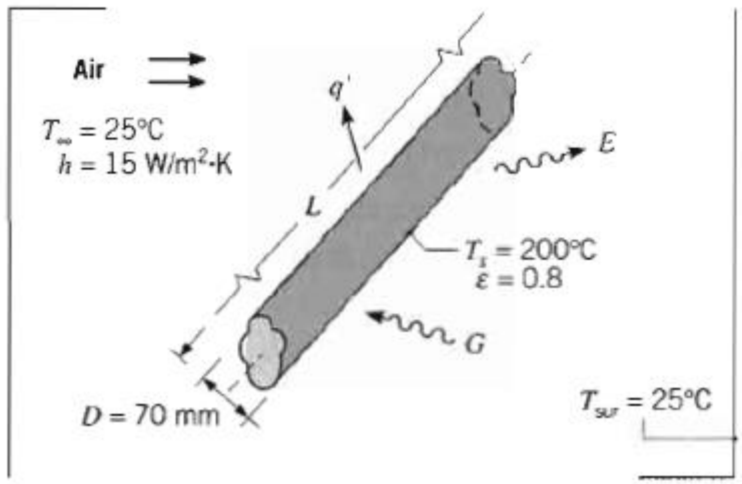
$$q''_x = -k\frac{T_2 - T_1}{L}$$

$$q''_x = k\frac{\Delta T}{L} = 1.7\ \text{W/m}\cdot\text{K} \times \frac{250\ \text{K}}{0.15\ \text{m}} = 2833\ \text{W/m}^2$$

$$q_x = (HW)\,q''_x = (0.5\ \text{m} \times 1.2\ \text{m})\,2833\ \text{W/m}^2 = 1700\ \text{W}$$

Example problem:

An uninsulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200°C and 0.8, respectively. What are the surface emissive power and irradiation? If the coefficient associated with free convection heat transfer from the surface to the air is 15 $W/m^2 \cdot K$, what is the rate of heat loss from the surface per unit length of pipe?



The surface emissive power may be evaluated from Equation 1.5, while the irradiation corresponds to $G = \sigma T_{sur}^4$. Hence

$$E = \varepsilon \sigma T_s^4 = 0.8(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)(473\ K)^4 = 2270\ W/m^2$$

$$G = \sigma T_{sur}^4 = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (298\ K)^4 = 447\ W/m^2$$

Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, $q = q_{conv} + q_{rad}$ and from Equation 1.10, with $A = \pi D L$,

$$q = h(\pi D L)(T_s - T_\infty) + \varepsilon(\pi D L)\sigma(T_s^4 - T_{sur}^4)$$

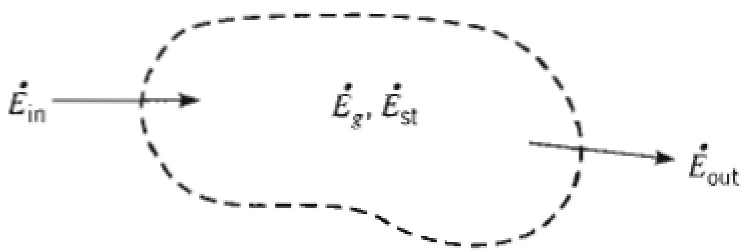
The heat loss per unit length of pipe is then

$$q' = \frac{q}{L} = 15\ W/m^2 \cdot K(\pi \times 0.07\ m)(200 - 25)°C$$

$$+ 0.8(\pi \times 0.07\ m)\ 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (473^4 - 298^4)\ K^4$$

$$q' = 577\ W/m + 421\ W/m = 998\ W/m$$

Conservation of Energy Principle:



Conservation of energy for control volume (CV) at an instant of time

Considering only the thermal energy one can state that:

The rate of increase of thermal energy in the CV must equal the rate at which thermal energy enters the CV, minus the rate at which thermal energy leaves the CV, plus the rate at which thermal energy is generated within the CV.

$$\Delta E_{st} = E_{in} - E_{out} + E_g$$

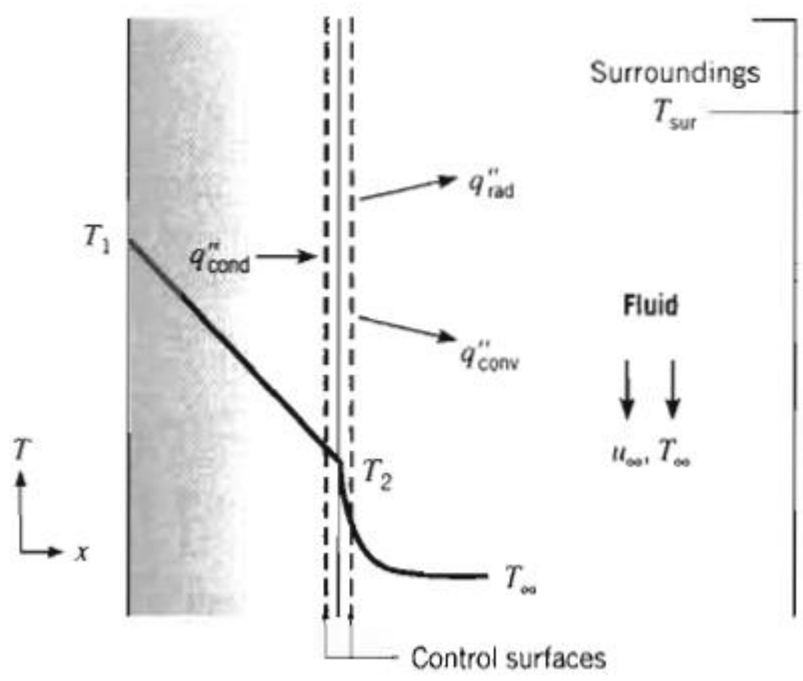
or

$$\dot{E}_{st} \equiv \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

The surface energy balance:

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_{cond} - q''_{conv} - q''_{rad} = 0$$



Example problem:

Consider a wall has temperatures 80 °C and 10 °C at surfaces and its thermal conductivity is 2 W/mK. Convective heat loss from the cold surface is 100 W/m^2. What is the thickness of this wall?

$$q''_{cond} - q''_{con} = 0$$

$$\frac{k}{L}(T_1 - T_2) = q''_{con} = 100\, W/m^2$$

$$L = \frac{k}{100}(T_1 - T_2) = \frac{2\, W/mK}{100\, W/m^2}(80 - 10)K \;= \mathbf{1.4\ m}$$

Example problem:

Consider a wall of perfectly insulated from one side and has 250 W/m^2 gain of heat from the other side. The surface gained heat is subjected to absolute vacuum condition and simultaneously losses heat to surroundings at -10 °C. Assume the surface is black. What is the surface temperature?

$$q''_{gain} - q''_{rad} = 0$$

$$250\, W/m^2 = \sigma(T_s^4 - T_{surr.}^4)$$

$$T_s = \left(\frac{250}{\sigma} + (263)^4\right)^{1/4} = \left(\frac{250}{5{,}67} 10^8 + (263)^4\right)^{1/4} = 309.65 \text{ K} = \mathbf{36.65\ °C}$$