For steady-state, two dimensional flow of an incompressible fluid with constant properties, the momentum equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

and the energy transfer equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\nu}{c_p}(\Phi + \dot{q})$$

where

$$\Phi = \left\{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]\right\}$$

At this stage, we need to add the mass transfer counterpart of the energy transfer equation. Before that some mass transfer related definitions are required.

Some useful definitions:

Mass density, ρ_i [kg/m³]: The amount of species i in a unit volume.

Molar concentration; C_i [kmol/m³]: The amount of mole of species i in a unit volume.

If M_i [kg/kmol] is the molar weight of species i, relation between ρ_i and C_i as follows: $\rho_i = M_i C_i$

ρ_i represents the mass of species i per unit volume of the mixture, then the mixture density,

$\rho = \sum \rho_i$ and $C = \sum C_i$

The amount of species i in the mixture may also be quantified in the mass fraction base as

$$m_i = \frac{\rho_i}{\rho}$$

and its mole fraction as

$$x_i = \frac{c_i}{c}$$

Then it fellows that $\Sigma m_i = 1$ and $\Sigma x_i = 1$

Heaving $M_i = \frac{\rho_i}{c_i}$ also leads $M = \frac{\rho}{c}$ for mixture

$$M = \frac{\rho}{c} = \Sigma \frac{M_i c_i}{c} = \Sigma x_i M_i \text{ also.}$$

Then now, the mas transfer equation, similar to the energy transfer equation,

$$u\frac{\partial m_i}{\partial x} + v\frac{\partial m_i}{\partial y} = D_{ij}\left(\frac{\partial^2 m_i}{\partial x^2} + \frac{\partial^2 m_i}{\partial y^2}\right)$$

or in mole fraction base,

$$u\frac{\partial c_i}{\partial x} + v\frac{\partial c_i}{\partial y} = D_{ij}\left(\frac{\partial^2 c_i}{\partial x^2} + \frac{\partial^2 c_i}{\partial y^2}\right)$$

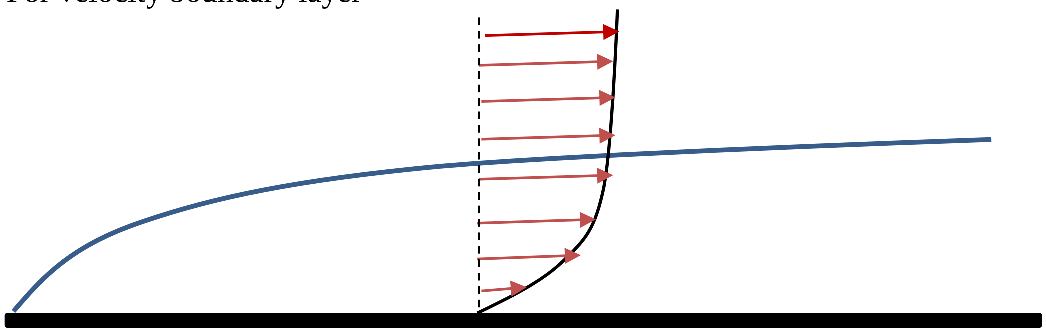
Here, D_{ij} is the mass diffusivity. It is the diffusion capacity of species i into j environment, like diffusion of water vapor (i) through dry air (j)

The boundary layer approximation and special conditions:

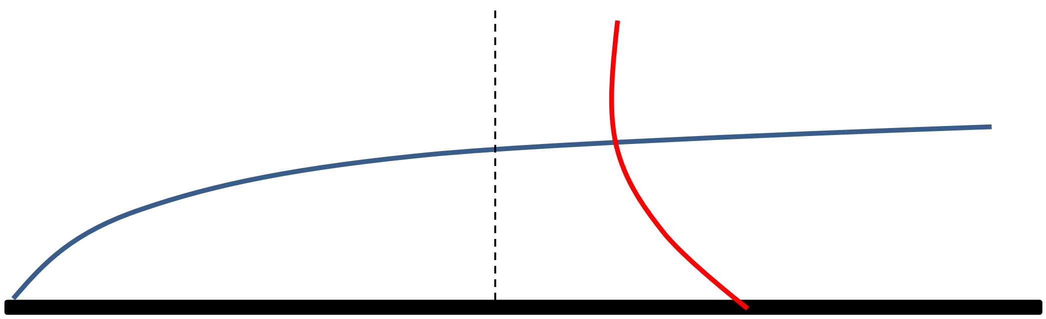
The following approximation can be made for boundary layer region.

Remarks on boundary layer??

For velocity boundary layer



For the temperature and/or species boundary layer



$$u \gg v; \ \frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \ \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$$

$$\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}$$

$$\frac{\partial m_i}{\partial y} \gg \frac{\partial m_i}{\partial x}$$

Then the equations becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= -\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial P}{\partial y}=0$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial m_i}{\partial x}+v\frac{\partial m_i}{\partial y}=D_{ij}\frac{\partial^2 m_i}{\partial y^2}$$

Sınır tabaka denklemlerinin boyutsuzlaştırılması; ısı ve kütle transferi boyutsuz parametreleri

$$x^* \equiv \frac{x}{L}$$

$$y^* \equiv \frac{y}{L}$$

$$u^* \equiv \frac{u}{u_\infty}$$

$$v^* \equiv \frac{v}{u_\infty}$$

$$p^* \equiv \frac{p}{\rho V^2}$$

$$T^* \equiv \frac{T-T_s}{T_\infty - T_s}$$

$$m_i^* \equiv \frac{m_i - m_{i,s}}{m_{i,\infty} - m_{i,s}}$$

Boyutsuz parametreler ile boyutsuz sınır tabaka denklemleri;

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{\nu}{u_\infty L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\Rightarrow 1/Re_L$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{\alpha}{u_\infty L}\frac{\nu}{\nu}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\Rightarrow \frac{\nu}{u_\infty L}\frac{\alpha}{\nu}=\frac{1}{Re_L}\frac{1}{Pr}$$

$$u^*\frac{\partial m_1^*}{\partial x^*}+v^*\frac{\partial m_1^*}{\partial y^*}=\frac{D_{ij}}{u_\infty L}\frac{\nu}{\nu}\frac{\partial^2 m_1^*}{\partial y^{*2}}$$

$$\Rightarrow \frac{\nu}{u_\infty L}\frac{D_{ij}}{\nu}=\frac{1}{Re_L}\frac{1}{Sc}$$

Yeni bir boyutsuz sayı, Schmidt sayısı; is the ratio of momentum diffusivity to mass diffusivity

$$Sc=\frac{\nu}{D_{ij}}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Re_L}\frac{1}{Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$u^* \frac{\partial m_1^*}{\partial x^*} + v^* \frac{\partial m_1^*}{\partial y^*} = \frac{1}{Re_L} \frac{1}{Sc} \frac{\partial^2 m_1^*}{\partial y^{*2}}$$

Then the functional relations for heat and mass transfer:

$Nu_x = f(x^*, Re_x, Pr)$, local

$Nu = f(Re_x, Pr)$, average

$Sh_x = f(x^*, Re_x, Sc)$, local

$Sh = f(Re_x, Sc)$, average

- Convective Mass Transfer

To relate it with convection heat transfer:

$$J_i'' = \rho h_m (m_{is} - m_{i\infty})$$

Mol fraction,

$$m_{is} = \frac{\rho_i}{\rho}$$

Substituting into mass transfer equation,

$$J_i'' = h_m (\rho_{is} - \rho_{i\infty})$$

- To determine densities, we treat the vapor as ideal gas;

Recalling definition of relative humidity (RH),

$$\emptyset = \frac{m_v}{m_g},$$

m_v: Vapor mass within the air

m_g: Vapor mass can be holded by the ait at existing temperature

From the ideal gas realation;

$$m_v = \frac{P_v V}{R_v T}$$

$$m_g = \frac{P_g V}{R_v T}$$

Evaluating with RH definition,

$$\emptyset = \frac{P_v;\, partial\ pressure\ of\ vapor}{P_g;\, satutration\ pressure\ of\ water\ at\ the\ existing\ themperature}$$

Having,

$$\rho = \frac{P}{RT}$$

Put them into convection mass transfer expression,

$$\boldsymbol{J_i^{"} = \frac{h_m}{R_v}\left(\frac{p_{vs}}{T_s} - \frac{p_{v\infty}}{T_\infty}\right)}.$$