EQUILIBRIUM IN THREE DIMENSIONS

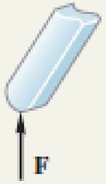
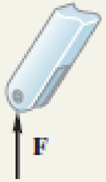
5.6. Conditions for Rigid-Body Equilibrium

The first step in solving three-dimensional equilibrium problems, as in the case of two dimensions, is to draw a free-body diagram. Before we can do this, however, it is first necessary to discuss the types of reactions that can occur at the supports.

Support Reactions

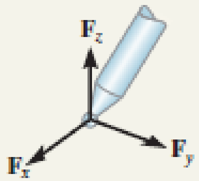
As in the two-dimensional case:

- A force is developed by a support that restricts the translation of its attached member.
- A couple moment is developed when rotation of the attached member is prevented.

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(4)

ball and socket



Three unknowns. The reactions are three rectangular force components.

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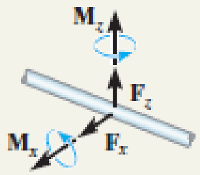
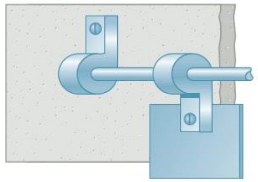
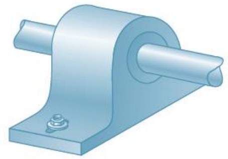
(5)

single journal bearing



Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.





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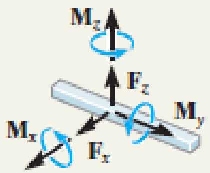
(6)

single journal bearing with square shaft



Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

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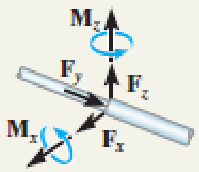
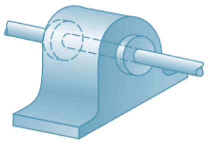
(7)

single thrust bearing

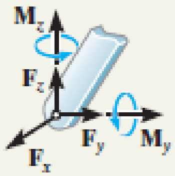


Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.



(8)

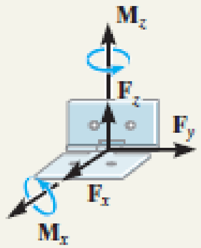
single smooth pin



Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

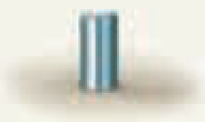
(9)

single hinge

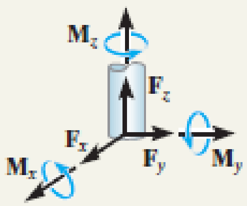


Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

(10)



fixed support



Six unknowns. The reactions are three force and three couple-moment components.

Free-Body Diagrams.

The general procedure for establishing the free-body diagram of a rigid body is the same as given in coplanar equilibrium.

NOTES:

- No equilibrium problem should be solved without first drawing the free-body diagram, so as to account for all the forces and couple moments that act on the body.
- If a support prevents translation of a body in a particular direction, then the support, when it is removed, exerts a force on the body in that direction.
- If rotation is prevented, then the support, when it is removed, exerts a couple moment on the body.
- Internal forces are never shown on the free-body diagram since they occur in equal but opposite collinear pairs and therefore cancel out.
- The weight of a body is an external force, and its effect is represented by a single resultant force acting through the body's center of gravity ***G***.
- Couple moments can be placed anywhere on the free-body diagram since they are free vectors. Forces can act at any point along their lines of action since they are sliding vectors.

5.7. Equations of Equilibrium

As stated in Sec. 5.1, the conditions for equilibrium of a rigid body subjected to a three-dimensional force system require that both the resultant force and resultant couple moment acting on the body be equal to zero.

$$\vec{F}_R = \sum \vec{F} = 0$$

$$(\vec{M}_R)_O = \sum \vec{M}_O = 0$$

If all the external forces and couple moments are expressed in Cartesian vector form

$$\sum \vec{F} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k}$$

$$\sum \vec{M}_O = \sum M_x \vec{i} + \sum M_y \vec{j} + \sum M_z \vec{k}$$

Since the i, j, and k components are independent from one another, the above equations are satisfied provided

$$\begin{array}{ll} \sum F_x = 0 & \sum M_x = 0 \\ \sum F_y = 0 & \sum M_y = 0 \\ \sum F_z = 0 & \sum M_z = 0 \end{array}$$

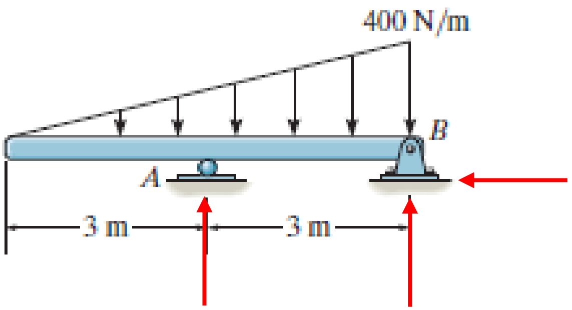
These six scalar equilibrium equations may be used to solve for at most six unknowns shown on the free-body diagram.

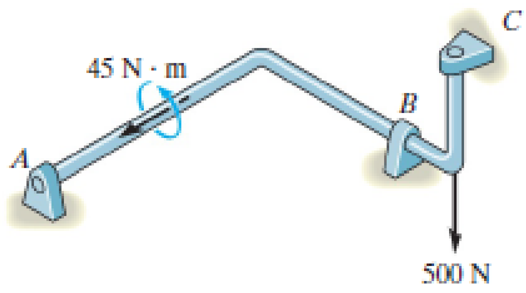
5.8. Constraints and Statically Determinacy

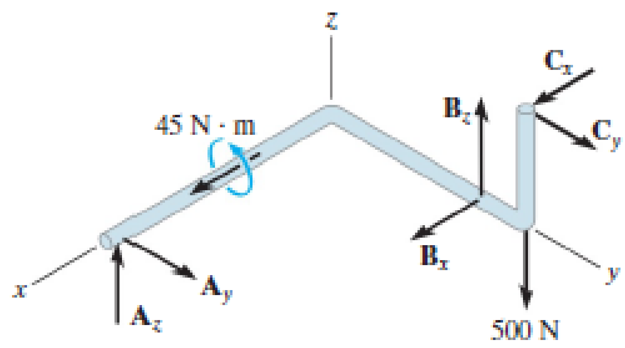
To ensure the equilibrium of a rigid body, it is not only necessary to satisfy the equations of equilibrium, but the body must also be properly held or constrained by its supports.

Statically determinacy.

If a body or system is constrained such a way that the number of unknowns for support reactions is equal to the number of equilibrium equations, then it becomes **statically determinate** or **isostatic**.





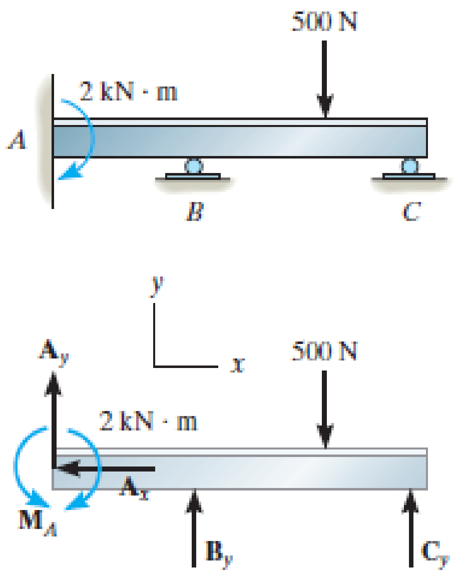


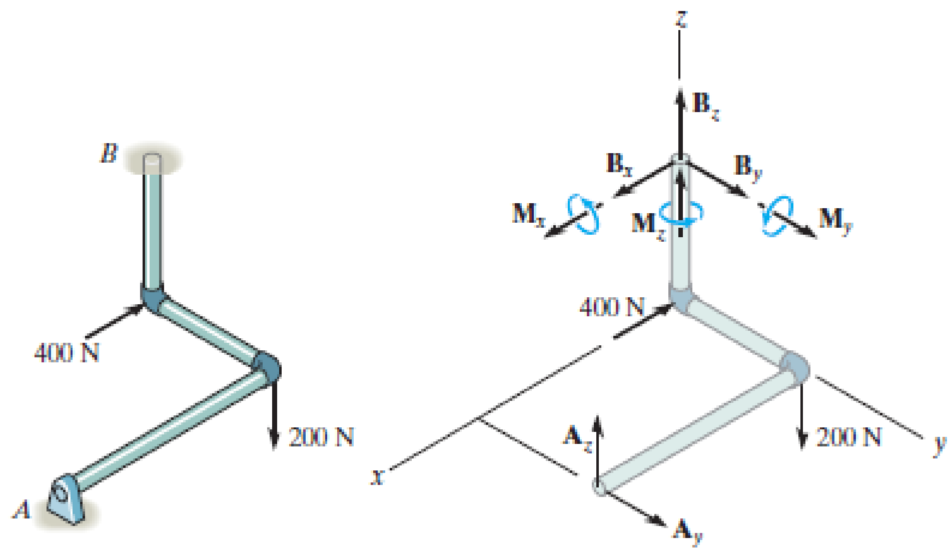
The force reactions developed by the bearings are sufficient for equilibrium since they prevent the shaft from rotating about each of the coordinate axes. No couple moments at each bearing are developed.

Redundant Constraints.

When a body has redundant supports, that is, more supports than are necessary to hold it in equilibrium, it becomes statically indeterminate. Statically indeterminate means that there will be more unknowns for support reactions on the body than equations of equilibrium available for their solution.

The additional equations are necessary to solve statically indeterminate problems, and they are generally obtained from the deformation conditions at the points of support. These equations involve the physical/material properties of the body which are studied in subjects dealing with the mechanics of deformation, such as mechanics of materials or strength of materials.

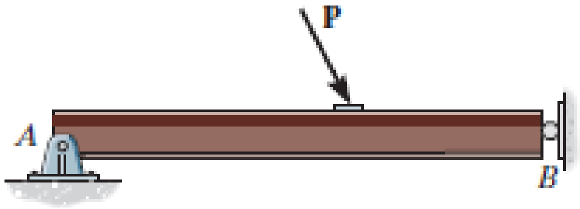


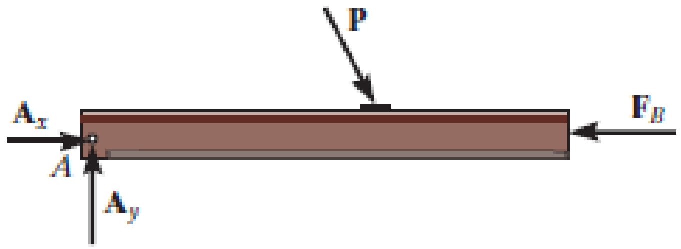


Improper constraints.

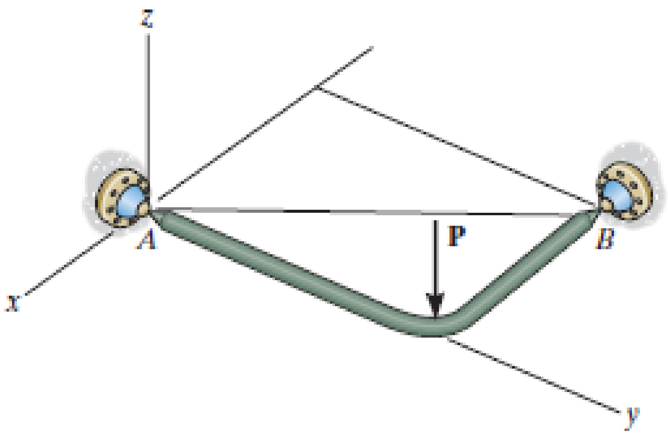
Having the same number of unknown reactive forces as available equations of equilibrium does not always guarantee that a body will be stable when subjected to a particular loading.

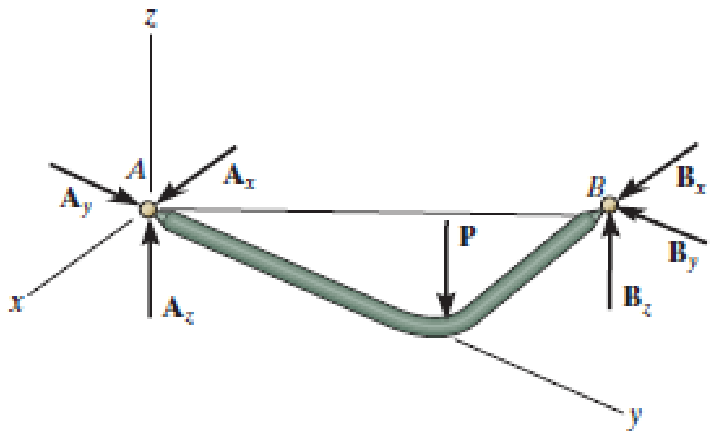
In the example, the lines of action of the reactive forces are concurrent at point A. Consequently, the applied loading P will cause the beam to rotate slightly about A. Thus, the moment equation of equilibrium about A is not satisfied, i.e., $\sum M_A \neq 0$.





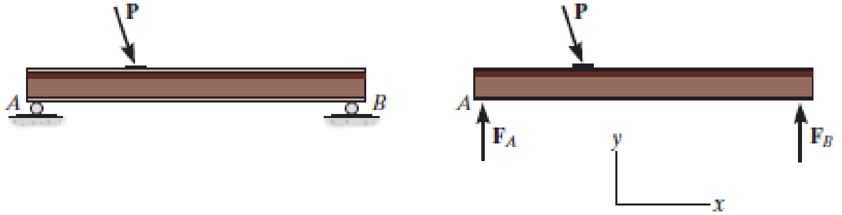
In three dimensions, the body will be improperly constrained if the lines of action of all the reactive forces intersect a common axis. The body shown in the figure will rotate about the AB, i.e., $\sum M_{AB} \neq 0$.



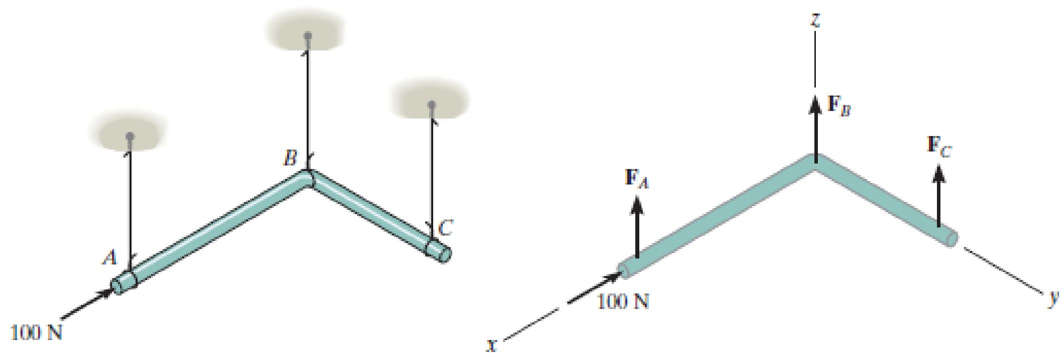


Another way in which improper constraining leads to instability occurs when the reactive forces are all parallel or a body may have fewer reactive forces than equations of equilibrium that must be satisfied.

Two- and three-dimensional examples of this are shown in figures. In both cases, the summation of forces along the x axis will not equal zero, i.e., $\sum F_x \neq 0$.



(a)



(b)

To summarize these points, a body is considered *improperly constrained:*

- If all the reactive forces intersect at a common point or pass through a common axis.
- If all the reactive forces are parallel.

In engineering practice, these situations should be avoided at all times since they will cause an unstable condition.

Procedure of Analysis

i. Establish the x, y and z axis in any suitable orientation. Choosing a good orientation makes the solution simpler.

ii. Remove all supports and draw FBD of the body. Show all the forces and couple moments acting on the body. Label all the loadings and specify their directions. The sense of a force or couple moment having an unknown magnitude but known line of action can be assumed.

iii. Indicate the dimensions of the body necessary for computing the moments of forces.

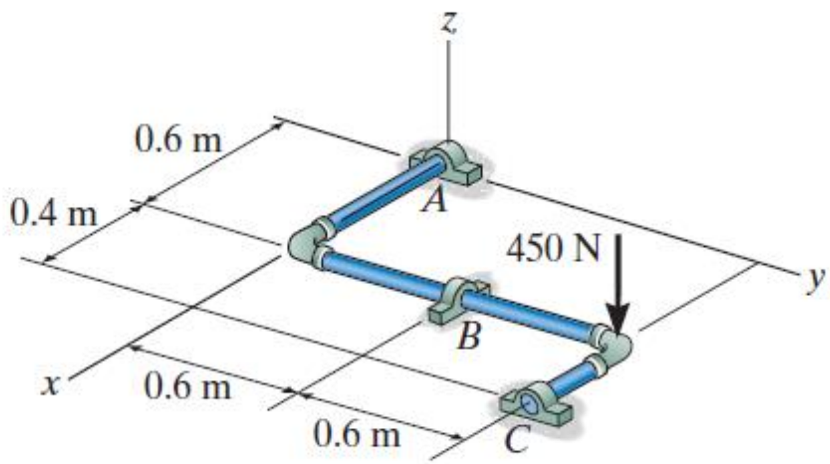
iv. If the x, y, z force and moment components seem easy to determine, then apply the six scalar equations of equilibrium; otherwise use the vector equations.

v. Choose the direction of an axis for moment summation such that it intersects the lines of action of as many unknown forces as possible. Realize that the moments of forces passing through points on this axis and the moments of forces which are parallel to the axis will then be zero.

vi. If the solution of the equilibrium equations yields a negative scalar for a force or couple moment magnitude, this indicates that the sense is opposite to that which was assumed on the free-body diagram.

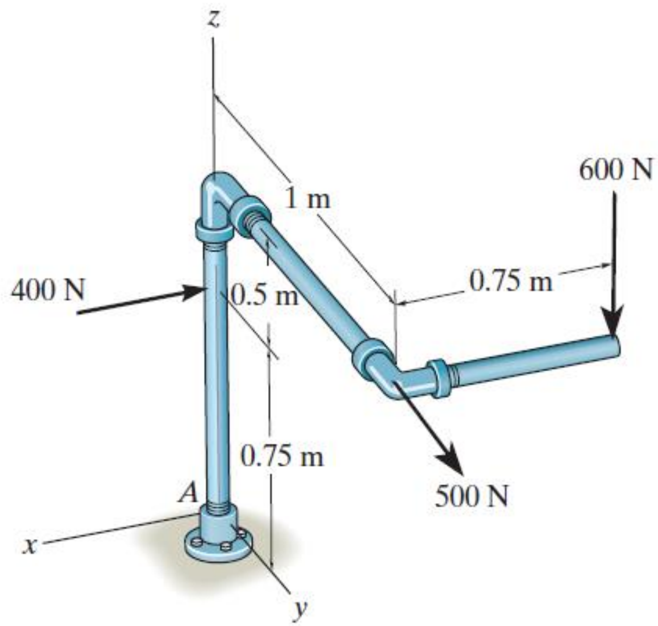
Example 7.

Determine the support reactions at the smooth journal bearings A, B, and C of the pipe assembly.



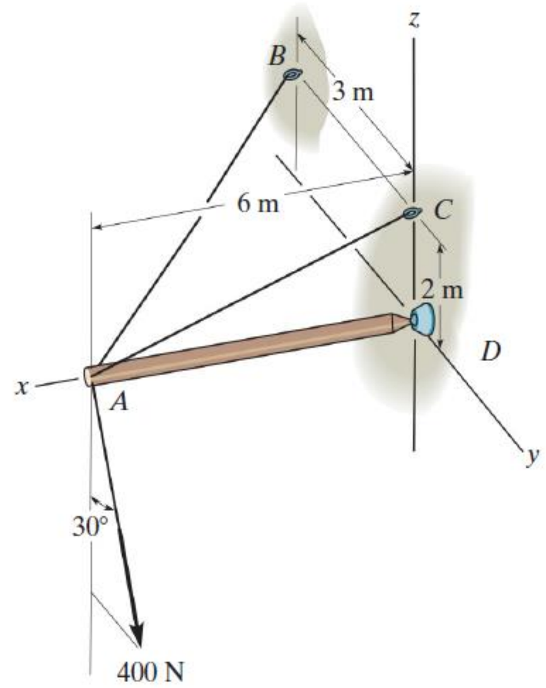
Example 8.

Determine the components of reaction at the fixed support A. The 400 N, 500 N, and 600 N forces are parallel to the x, y, and z axes, respectively.



Example 9.

Determine the tension in each cable and the components of reaction at D needed to support the load.



Example 10.

Determine the reactions at the roller support A, the ball-and-socket joint D, and the tension in cable BC for the plate.