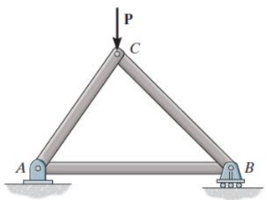
CHAPTER 6. STRUCTURAL ANALYSIS:TRUSSES

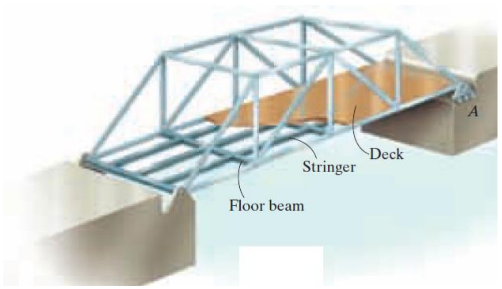
COPLANAR (2D) TRUSSES

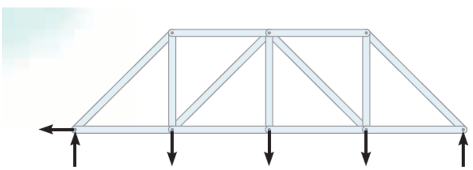
6.1 . Introduction



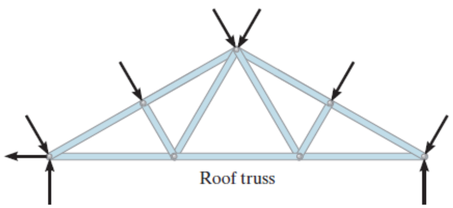
The problems considered in the preceding chapters concerned the equilibrium of a single rigid body, and all forces involved were external to the rigid body. We now consider problems dealing with the equilibrium of structures made of several connected parts. These
problems call for the determination not only of the external forces acting on the structure but also of the forces which hold together the various parts of the structure. From the point of view of the structure as a whole, these forces are **internal forces**.

A **truss** is a structure composed of slender members joined together at their end points. In particular, planar trusses lie in a single plane and are often used to support roofs and bridges.





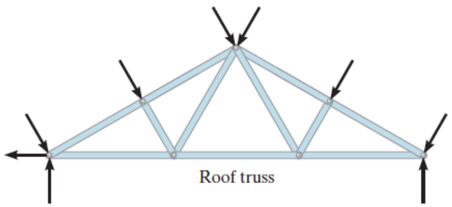
Bridge truss



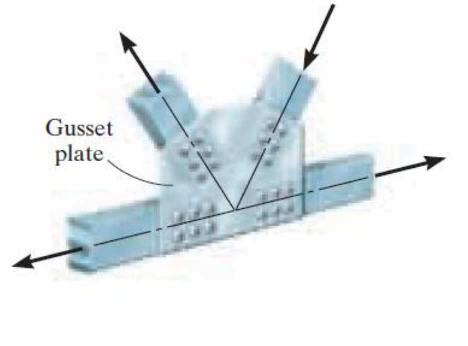
Roof truss

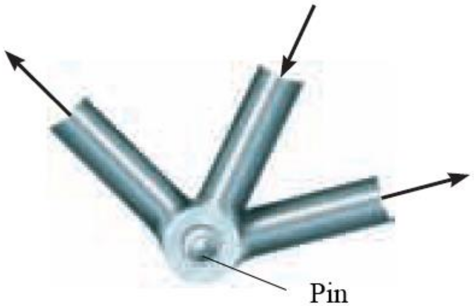
(b)

Assumptions for Design.



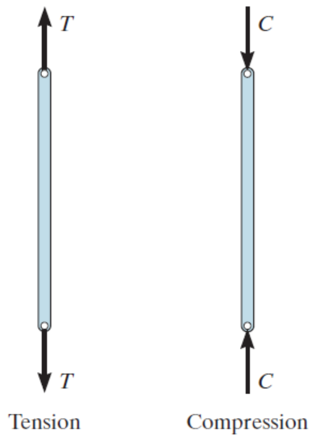
(b)





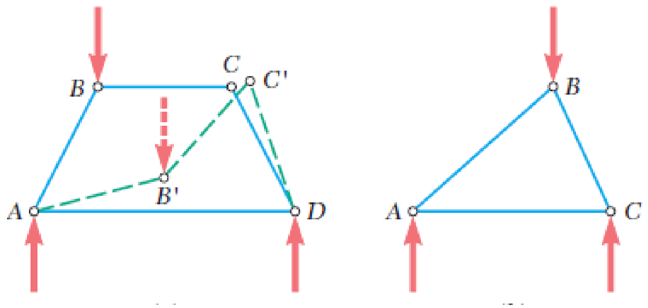
i) *All loadings are applied at the joints*. In most situations, such as for bridge and roof trusses, this assumption is true. Frequently the weight of the members is neglected because the force supported by each member is usually much larger than its weight. However, if the weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.

ii) *The members are joined together by smooth pins*. The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate, or by simply passing a large bolt or pin through each of the members. We can assume these connections act as pins provided the center lines of the joining members are concurrent. In other words, there will be no moment at the joint connections, only forces are produced.

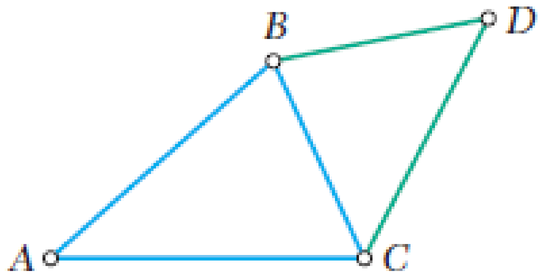


Because of these two assumptions, each truss member will act as a two-force member, and therefore the force acting at each end of the member will be directed along the axis of the member. If the force tends to elongate the member, it is a tensile force (T), whereas if it tends to shorten the member, it is a compressive force (C). In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive. Often, compression members must be made thicker than tension members because of the buckling or column effect that occurs when a member is in compression.

Simple Truss.

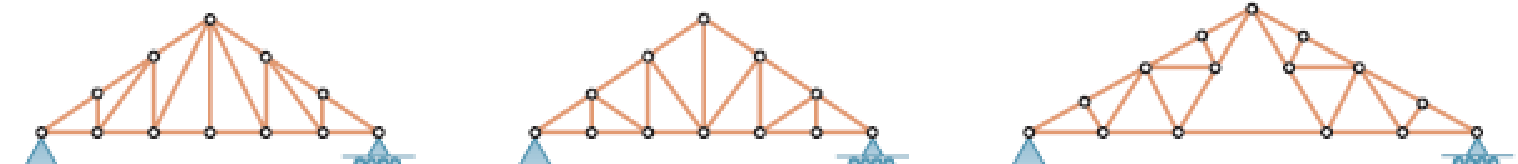


Consider the truss in the figure, which is made of four members connected by pins at A, B, C, and D. If a load is applied at B, the truss will greatly deform, completely losing its original shape. In contrast, the truss which is made of three members connected by pins at A, B, and C, will deform only slightly under a load applied at B. The only possible deformation for this truss is one involving small changes in the length of its members. Three member truss is said to be a **rigid truss**, the term rigid being used here to indicate that the truss will not collapse.

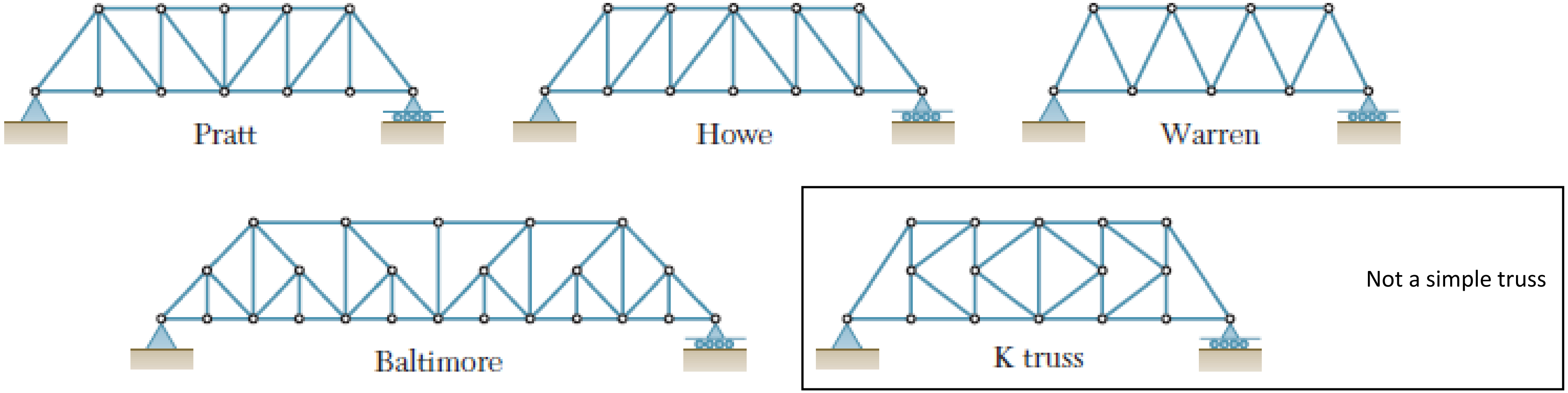


As shown in the figure, a larger rigid truss can be obtained by adding two members BD and CD to the basic triangular truss. This procedure can be repeated as many times as desired, and the resulting truss will be rigid if each time two new members are added, they are attached to two existing joints and connected at a new joint. A truss which can be constructed in this manner is called a **simple truss**.

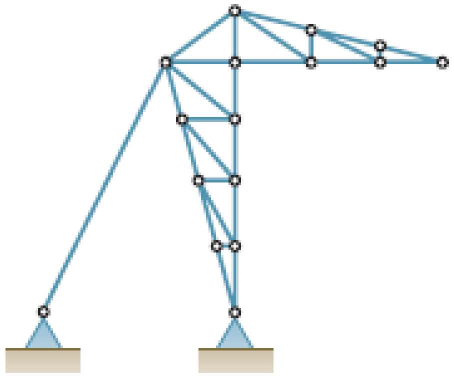
Two methods can be used in the analysis of truss structures. They are **the method of joints** and **the method of sections**. Each of the methods has its own advantages and disadvantages. In some problems both methods can be used for simpler analysis.



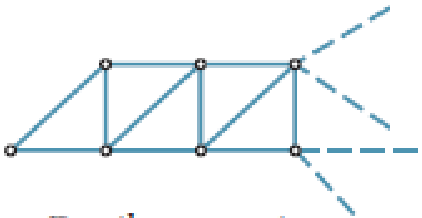
Typical Roof Trusses



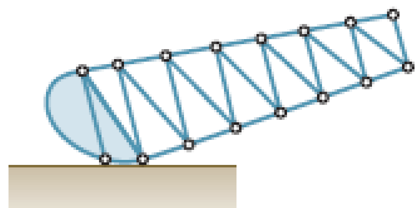
Typical Bridge Trusses



Stadium

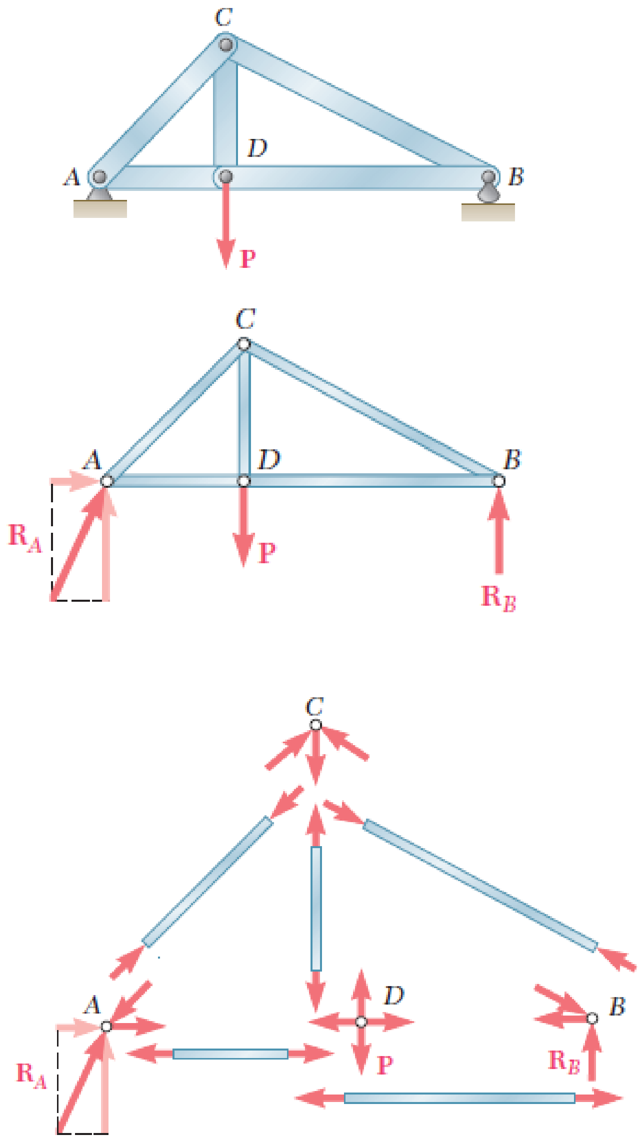


Cantilever portion of a truss



Bascule

6.2 The Method of Joints



A truss can simply be considered as a group of joints and two-force members. This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium. Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint. Since the members of a plane truss are straight two-force members lying in a single plane, each joint is subjected to a force system that is coplanar and concurrent. As a result, only $F_x = 0$ and $F_y = 0$ need to be satisfied for equilibrium (2 equilibrium equations for each joint).

For example, in the figure, The truss can thus be dismembered, and a free-body diagram can be drawn for each pin and each member. Each member is acted upon by two forces, one at each end; these forces have the same magnitude, same line of action, and opposite sense (two force member). Furthermore, Newton's third law indicates that the forces of action and reaction between a member and a pin are equal and opposite. Therefore, the forces exerted by a member on the two pins it connects must be directed along that member and be equal and opposite.

Important Points.

i) Simple trusses are composed of triangular elements. The members are assumed to be pin connected at their ends and loads applied at the joints.

ii) If a truss is in equilibrium, then each of its joints is in equilibrium. The internal forces in the members become external forces when the free-body diagram of each joint of the truss is drawn. A force pulling on a joint is caused by tension in a member, and a force pushing on a joint is caused by compression.

Procedure for Analysis

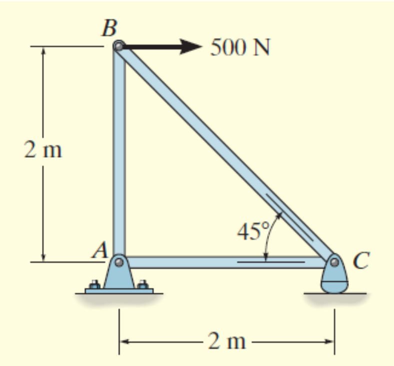
i) Find support reactions assuming whole structure is a rigid body (if necessary or asked).

ii) Draw the free-body diagram of the joints. Start from the joints that have minimum number unknown forces (member forces+support reactions if not found). Assume the sense of the force in the members as tension or compression. (***Advice****: assume all the unknown member forces as tension (positive) at first*)

iii) In each joint, orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations $F_x = 0$ and $F_y = 0$. Solve for the two unknown member forces and verify their correct sense.

iv) If the solution of the equilibrium equations yields a negative scalar for a member force, this indicates that the sense is opposite to that which was assumed on the free-body diagram. (If you assume all the unknown member forces as tension, a positive ssolution is meant a tension force whereas a negative solution is meant a compressive force)

Application of the Procedure



i) Find support reactions

(FBD)

$\xrightarrow{+}\ \sum F_x = 0;\quad A_x + 500 = 0 \;\rightarrow\; A_x = -500\ N\ (\leftarrow)$

$\circlearrowleft +\ \sum M_A = 0;\quad -500*2 + C*2 = 0 \;\rightarrow\; C = 500\ N$

$+\uparrow\ \sum F_y = 0;\quad A_y + C = 0 \;\rightarrow\; A_y = -500\ N\ (\downarrow)$

(FBD with correct directions)

ii) draw free-body diagrams for each joint

(FBD of A) (FBD of B) (FBD of C)

iii) apply equations of equilibrium

$\xrightarrow{+}$ $\sum F_x = 0; \quad S_2 - 500 = 0 \quad \rightarrow \quad S_2 = 500\ N \quad (T)$

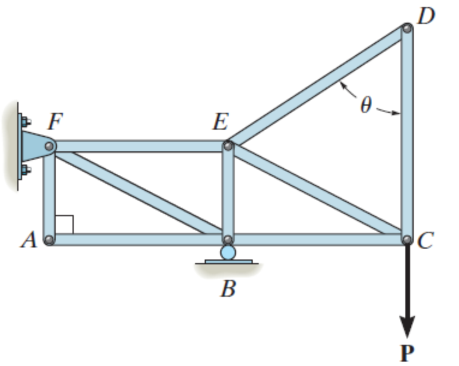
$+\uparrow$ $\sum F_y = 0; \quad S_1 - 500 = 0 \quad \rightarrow \quad S_1 = 500\ N \quad (T)$

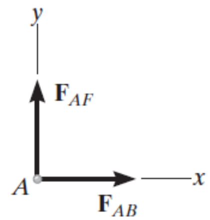
$+\uparrow$ $\sum F_y = 0;$

$S_3 \sin 45 + 500 = 0$

$\rightarrow \quad S_3 = -\dfrac{500}{\sin 45} = -707.12\ N \quad (C)$

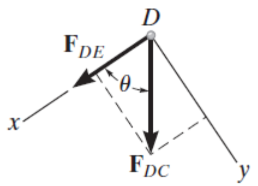
6.3 . Zero Force Members





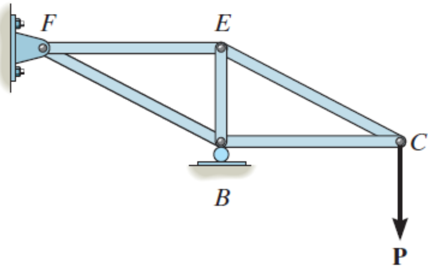
$\xrightarrow{+}\ \Sigma F_x = 0;\ F_{AB} = 0$

$+\uparrow\ \Sigma F_y = 0;\ F_{AF} = 0$



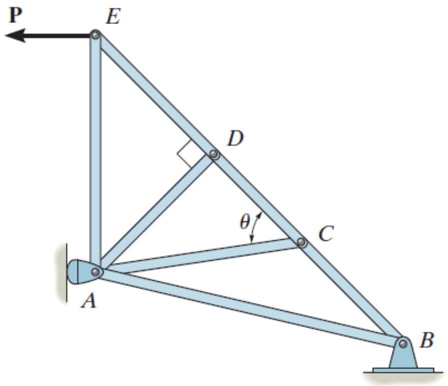
$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\quad F_{DC} = 0 \text{ since } \sin\theta \neq 0$

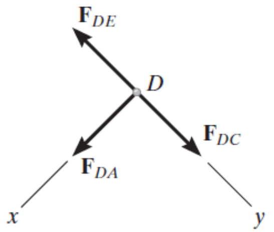
$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\quad F_{DE} = 0$



Truss analysis using the method of joints is greatly simplified if we can first identify those members which support no loading. These zero-force members are used to increase the stability of the truss during construction and to provide added support if the loading is changed.

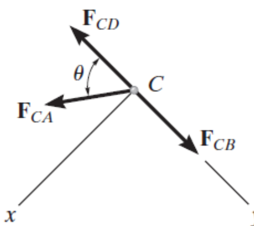
The zero-force members of a truss can generally be found by inspection of each of the joints. For example, consider the truss shown in the figure. If a free-body diagram of the pin at joint A is drawn, it is seen that members AB and AF are zero-force members. (We could not have come to this conclusion if we had considered the free-body diagrams of joints F or B simply because there are five unknowns at each of these joints.) In a similar manner, consider the free-body diagram of joint D. Here again it is seen that DC and DE are zero-force members. From these observations, we can conclude that *if only two non-collinear members form a truss joint and no*





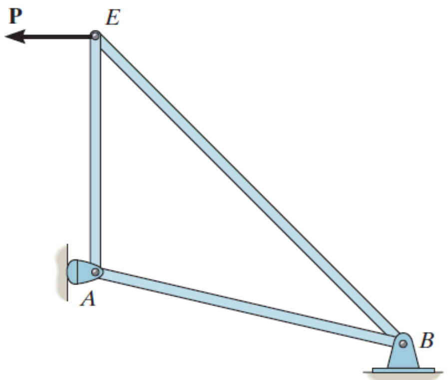
$+\swarrow \Sigma F_x = 0; \quad F_{DA} = 0$

$+\searrow \Sigma F_y = 0; \quad F_{DC} = F_{DE}$



$+\swarrow \Sigma F_x = 0; \quad F_{CA} \sin\theta = 0; \quad F_{CA} = 0 \text{ since } \sin\theta \neq 0;$

$+\searrow \Sigma F_y = 0; \quad F_{CB} = F_{CD}$

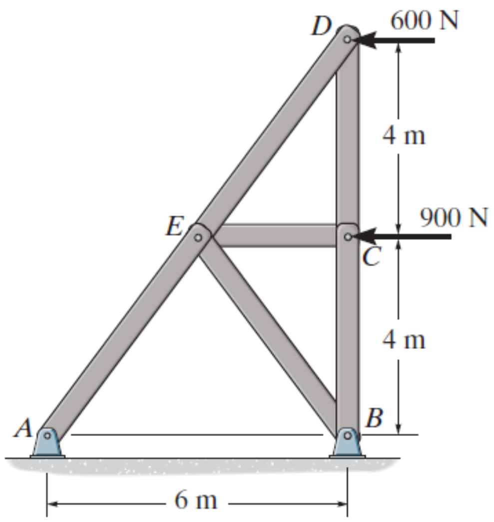


external load or support reaction is applied to the joint, the two members must be zero-force members.

Now consider the truss shown in the figure. The free-body diagram of the pin at joint D is shown. By orienting the y axis along members DC and DE and the x axis along member DA, it is seen that DA is a zero-force member. Note that this is also the case for member C. In general then, *if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction has a component that acts along this member*.

Example 1.

Determine the force in each member of the truss and state if the members are in tension or compression.



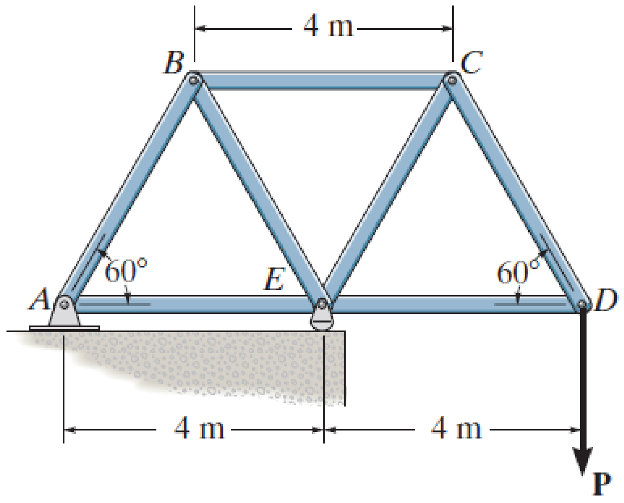
i) no need to find support reactions

ii) FBD of each joint & equilibrium equations

Example 2.

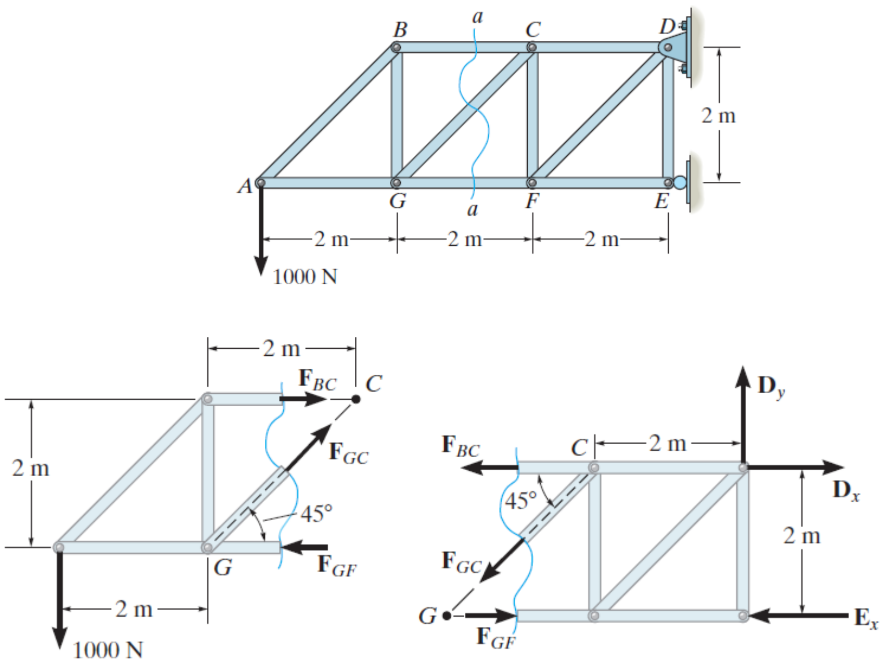
If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force P that can be supported at joint D.

(Each member has same length)



6.4 The Method of Sections

The method of joints is most effective when the forces in all the members of a truss are to be determined. If, however, the force in only one member or the forces in a very few members are desired, another method, **the method of sections**, is more efficient. It is based on the principle that *if the truss is in equilibrium then any segment of the truss is also in equilibrium*.



For example, consider the truss in the figure. If the forces in members BC, GC, and GF are to be determined, then section aa would be appropriate. The free-body diagrams of the two segments are shown at the bottom. Note that the line of action of each member force is specified from the geometry of the truss, since the force in a member is along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part—Newton's third law. Members BC and GC are assumed to be in tension since they are subjected to a "pull," whereas GF in compression since it is subjected to a "push." The wanted member forces can be found from left or right part of the truss. At the left, F_{BC}, F_{GC}, and F_{GF} are the only unknowns (3 unknowns). Applying equilibrium equations (3 eqs.), unknown member forces can be found. However at the right part, in addition to F_{BC}, F_{GC}, and F_{GF} , support reactions are also unknowns (total 6 unknowns).

Therefore, first, the reaction forces should be found by the help of applying equilibrium equations to the whole system, then unknown member forces can be found from the equililibrium of right part.

Procedure for Analysis

i) Make a decision on how to “cut” or section the truss through the members where forces are to be determined (i & ii can change order).

ii) Find support reactions assuming whole structure is a rigid body (if necessary).

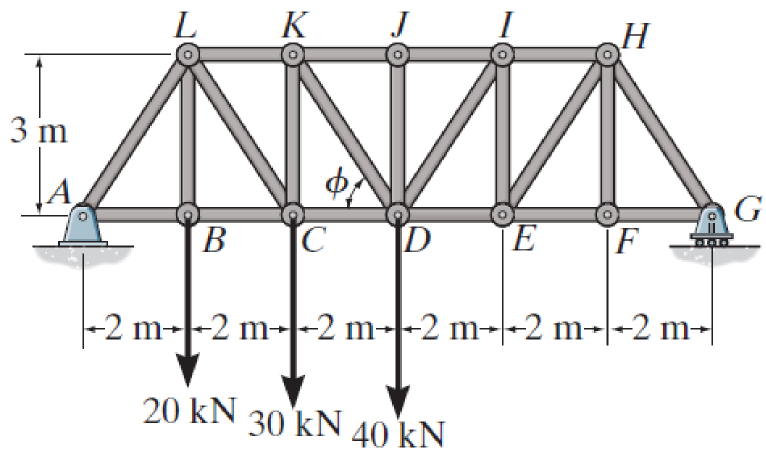
ii) Draw the free-body diagram of the segment which has the least number of forces acting on it (it is an advice not a obligation).

iii) Apply equations of equilibrium and find unknown member forces.

NOTE: If necessary, more than one section can be used in one problem or the method of joints and sections can be used simultaneously.

Example 3.

Determine the force in members LK, KC, and CD of the Pratt truss. State if the members are in tension or compression.



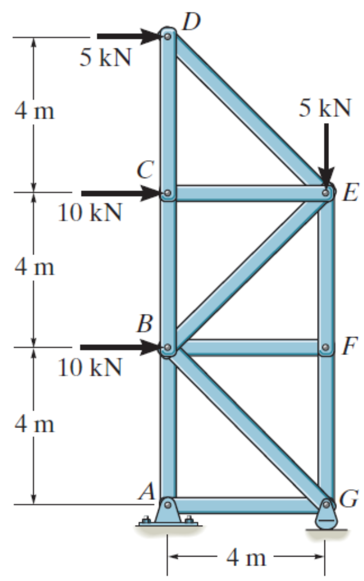
Example 4.

Determine the force in members *BF*, *BG*, and *AB*,

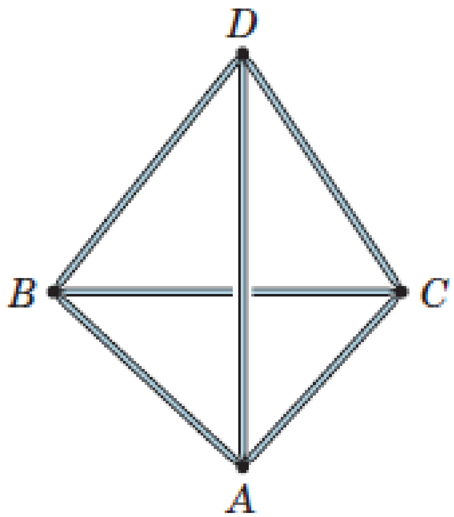
and state if the members are in tension or compression.

i) no need support reactions

ii) cutting



6.5 Space (3D) Trusses



A **space truss** consists of members joined together at their ends to form a stable three-dimensional structure. The simplest form of a space truss is a *tetrahedron*, formed by connecting six members together, as shown in the figure. Any additional members added to this basic element would be redundant in supporting the force $\vec{P}$. A simple space truss can be built from this basic tetrahedral element by adding three additional members and a joint, and continuing in this manner to form a system of multi-connected tetrahedrons.

Assumptions for Design.

The members of a space truss may be treated as two-force members provided the external loading is applied at the joints and the joints consist of ball-and-socket connections (only forces not moments). These assumptions are justified if the welded or bolted connections of the joined members intersect at a common point and the weight of the members can be neglected. In cases where the weight of a member is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, half of its magnitude applied at each end of the member.

Procedure for Analysis

Method of Joints. If the forces in all the members of the truss are to be determined, then the method of joints is most suitable for the analysis. Here it is necessary to apply the three equilibrium equations $F_x = 0$, $F_y = 0$, $F_z = 0$ to the forces acting at each joint. Remember that the solution of many simultaneous equations can be avoided if the force analysis begins at the joint that has minimum number of unknown forces. Also, if the three-dimensional geometry of the force system at the joint is hard to visualize, it is recommended that a Cartesian vector analysis be used for the solution.

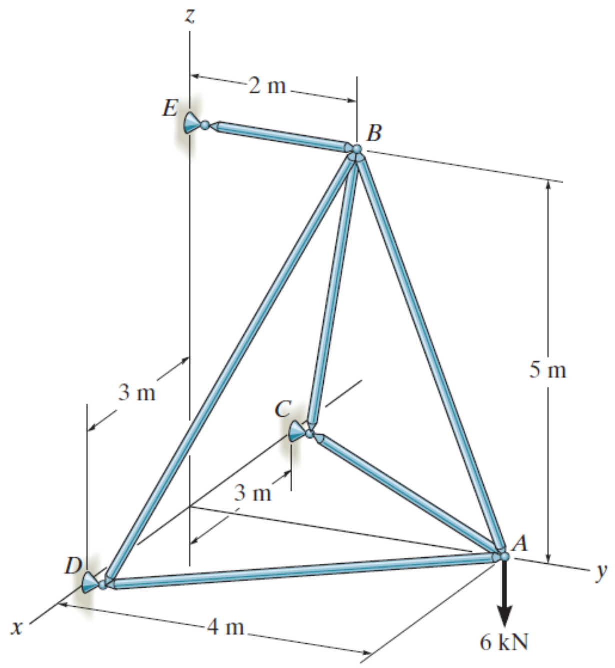
Method of Sections. If only a few member forces are to be determined, the method of sections can be used. When an imaginary section is passed through a truss and the truss is separated into two parts, the force system acting on one of the segments must satisfy the six equilibrium equations:

$F_x = 0$, $F_y = 0$, $F_z = 0$, $M_x = 0$, $M_y = 0$, $M_z = 0$

By proper choice of the section and axes for summing forces and moments, many of the unknown member forces in a space truss can be computed directly, using a single equilibrium equation.

Example 5.

Determine the force in each member of the space truss and state if the members are in tension or compression.



Example 6.

Determine the force in members EF, AF, and DF of the space truss and state if the members are in tension or compression. The truss is supported by short links at A, B, D, and E.