CHAPTER 3. EQUILIBRIUM OF A PARTICLE

3.1. Condition for the Equilibrium of a Particle

A particle is said to be in equilibrium if it remains at rest if originally at rest, or has a constant velocity if originally in motion. Most often, however, the term "equilibrium" or, more specifically, "static equilibrium" is used to describe an object at rest. To maintain equilibrium, it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on a particle to be equal to zero. This condition is stated by the equation of equilibrium,

$$\sum \vec{\mathbf{F}} = 0$$

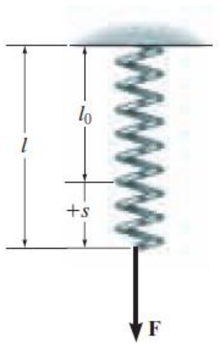
where F is the vector sum of all the forces acting on the particle.

3.2. Free-Body Diagram

In practice, a problem in engineering mechanics is derived from an actual physical situation. A sketch showing the physical conditions of the problem is known as a ***space* diagram**. A large number of problems involving actual structures, however, can be reduced to problems concerning the equilibrium of a particles by choosing one or more significant particles. To apply the equation of equilibrium, we must account for all the known and unknown forces ($\sum \vec{\mathbf{F}}$) which act on the particle. The best way to do this is to think of the particle as isolated and "free" from its surroundings. A drawing that shows the particle with all the forces that act on it is called a **free-body diagram** (**FBD**).

Before presenting a procedure how to draw a free-body diagram, we will first consider some of the support types encountered in particle equilibrium problems.

i. Springs.



If a linearly elastic spring (or cord) of undeformed length l_0 is used to support a particle, the length of the spring will change in direct proportion to the force $\vec{F}$ acting on it. A characteristic that defines the "elasticity" of a spring is the **spring constant** or **stiffness, k**.

The magnitude of force exerted on a linearly elastic spring which has a stiffness k and is deformed (elongated or compressed) a distance $s = l - l_0$, measured from its unloaded position, is

$$F = ks$$

If s is positive, causing an elongation, then F must pull on the spring; whereas if *s* is negative, causing a shortening, then F must push on it.

Consider a body (particle) attached to the tip of a spring, the spring tries to pull the body if $s>0$, whereas the spring tries to push the particle if $s<0$. A spring always want to go back to its original size, l_0.

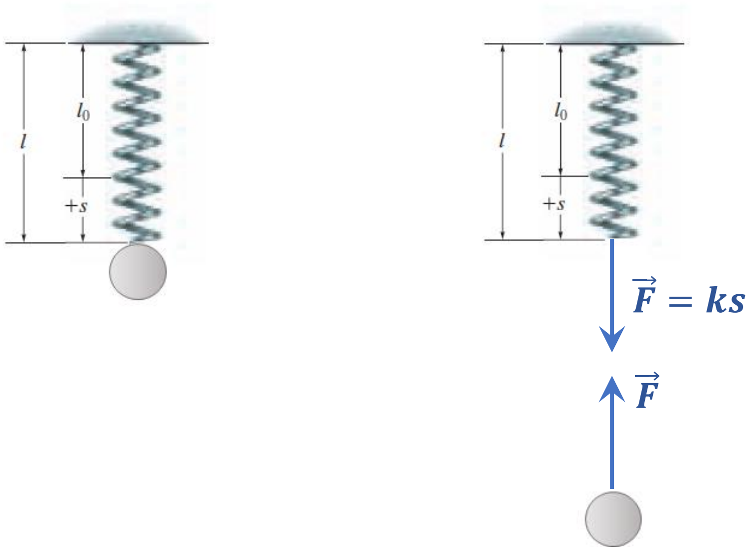


Fig. 3.1. A particle attached to a spring

ii. **Cables.**

Unless otherwise stated, all cables (or cords) will be assumed to have negligible weight and they cannot stretch. Also, a cable can support only a tension or "pulling" force, and this force always acts in the direction of the cable. A cable has a constant tension *T* throughout its length (same cable same force).

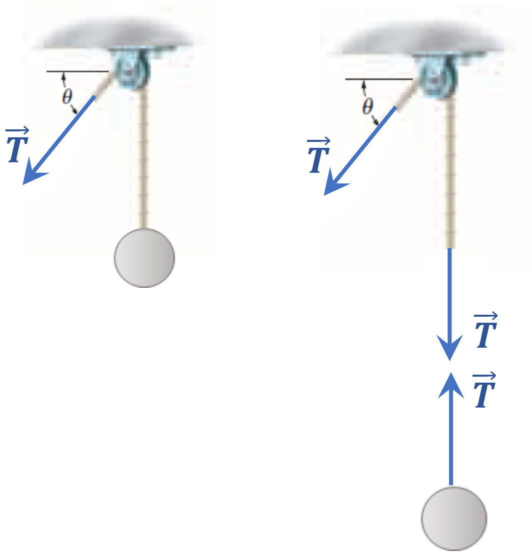


Fig. 3.2. A particle attached to a cable

iii. Smooth Surface Contact

If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface at the point of contact.

Fig. 3.3. A smooth surface contact

iv. Smooth Body Contact

If two or more bodies are in contact with each other, then they will apply equal forces to each other from the gravity center of one body to another.

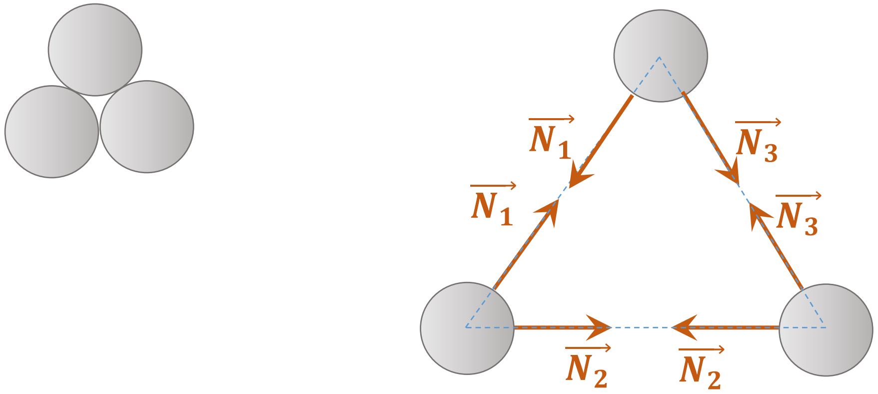


Fig. 3.4. A smooth body contact

Procedure of FBD

i. Choose one or more (enough) number of significant points (particles or bodies) in the system such that the problem can be reduced equilibrium of these selected points. These significant points are the intersection points of line of actions of forces in the system and they are generally center of gravities of rigid bodies or connection point of cables and springs.

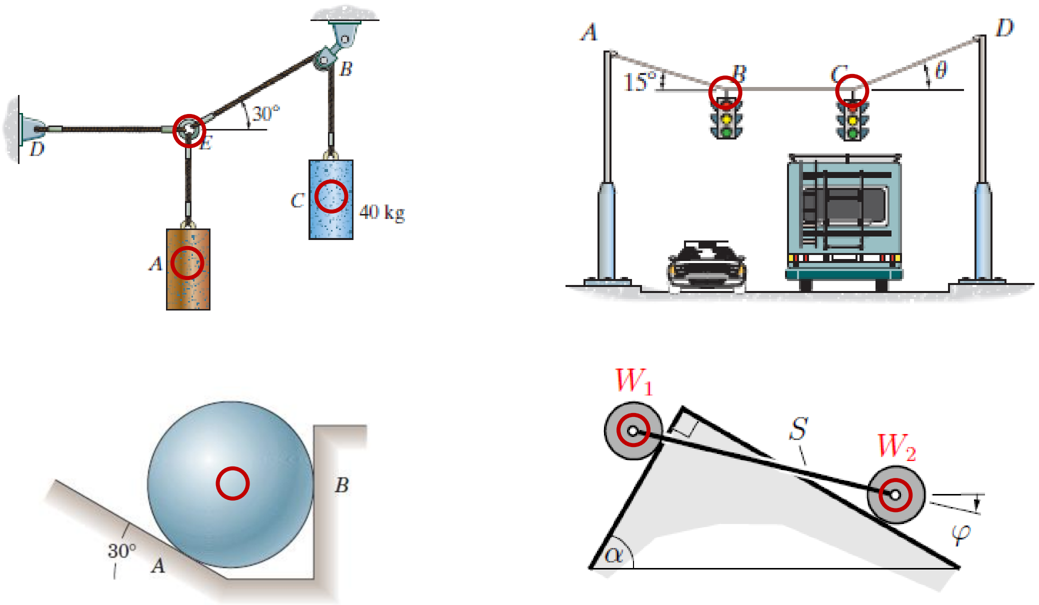
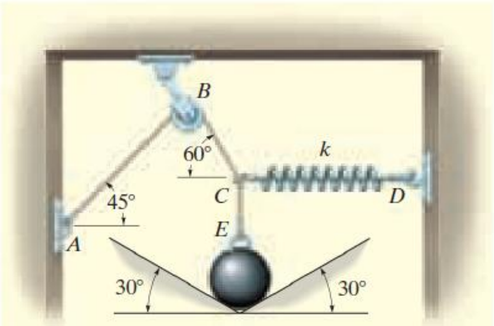


Fig. 3.5. Significant points in systems

ii. Imagine the particle(s) to be isolated or cut “free” from its surroundings. This requires removing all the supports (spring, cable, surface or another body) and drawing the particle(s) by itself. If there are more than one particle, each one of them should be drawn separately.

iii. Show all the forces (external forces applied to the system or the forces originate from removing the supports) acting on that particle on the drawing. The forces should be drawn with their proper magnitude and direction.

Example 3.1



Draw free body diagram of the following system.

Solution:

3.3. Coplanar Force Systems

If a particle is subjected to a system of coplanar forces that lie in the same plane, *e.g. xy* plane, then each force can be resolved into its *rectangular* components. For equilibrium, these forces must sum to produce a zero force resultant.

$$\vec{\mathbf{R}} = \sum \vec{\mathbf{F}} = \mathbf{0}$$

For example, for a force system that lies in a *xy* plane,

$$\sum \vec{\mathbf{F}} = \mathbf{0}$$

$$\sum F_x \vec{\mathbf{i}} + \sum F_y \vec{\boldsymbol{j}} = \mathbf{0}$$

For this vector equation to be satisfied, the resultant force components must both be equal to zero. Hence,

$$\xrightarrow{+} \quad \sum F_x = \mathbf{0} \qquad (3.1)$$

$$+\uparrow \quad \sum F_y = \mathbf{0} \qquad (3.2)$$

Since Eqs. (3.1 and 3.2) are addition of magnitudes of the forces (scalars), the algebraic signs of these scalars should be selected according to chosen direction. If the arrow head of a vector is towards the selected direction, it has a positive magnitude whereas if the arrow head of a vector is opposite to the selected direction, it has a negative magnitude. These two equations can be solved for at most two unknowns, generally represented as angles and magnitudes of forces shown on the particle's free-body diagram.

It is important to note that if a force has an unknown magnitude, then the arrowhead sense of the force on the free-body diagram can be assumed. Then if the solution yields a negative scalar, this indicates that the sense of the force is opposite to that which was assumed.

Problem Types

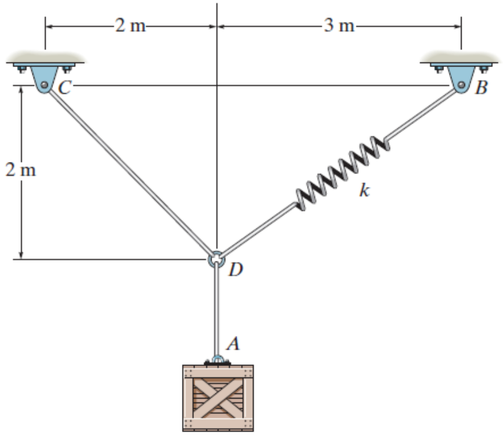
i. Control of equilibrium. All the forces are known in the system and controlling of the equilibrium is asked.
ii. Finding the unknown(s). Some of the quantities in the system has unknown(s) such as magnitude of a force, direction of a force, mass of a body, stiffness of a spring and force in cable which are asked to be found.
iii. Finding maximum value of a quantity. If a quantity of the system has a maximum value, maximum value of some other quantities such as magnitude of a force or mass of a body are asked.

Solution Procedure:

i. If not given in the problem, establish the x and y axis in any suitable orientation. Choosing a good orientation makes the solution simpler.
ii. Decide significant points (particles or bodies) of the problem.
iii. Draw FBD of each of the points. It should be noted that all the forces (external or support) should be drawn with their proper magnitude and direction. If there is a force having an unknown magnitude acting on the point, the sense of the force can be assumed.
iv. Apply the equations of equilibrium, $\sum F_x = \mathbf{0}$ and $\sum F_y = \mathbf{0}$, for each points. For convenience, arrows can be written alongside each equation to define the positive directions. Components are positive if they are directed along a positive direction, and negative if they are directed along a negative direction.
v. Since the magnitude of a force is always a positive quantity, then if the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

Example 3.2

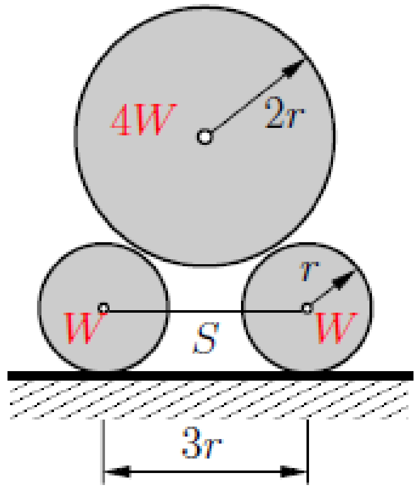
Determine the unstretched length of DB to hold the 40-kg crate in the position shown. Take $k = 180 N/m$.



Solution:

Example 3.3

A large cylinder (weight *4W*, radius *2r*) lies on top of two small cylinders (each having weight *W* and radius *r*) as shown in the figure. The small cylinders are connected by a wire S (length 3r). All surfaces are smooth. Determine all contact forces and the magnitude of force S in the wire.



3.4. Three-Dimensional Force Systems

Similar to coplanar force systems, for equilibrium, forces must sum to produce a zero force resultant.

$$\vec{\mathbf{R}} = \sum \vec{\mathbf{F}} = \mathbf{0}$$

For example, for a force system that lies in a *xyz* plane,

$$\sum \vec{\mathbf{F}} = \mathbf{0}$$

$$\sum F_x \vec{\mathbf{i}} + \sum F_y \vec{\mathbf{i}} + \sum F_z \vec{\mathbf{k}} = \mathbf{0}$$

For this vector equation to be satisfied, the resultant force components must both be equal to zero. Hence,

$$\overset{+}{\swarrow} \quad \sum F_x = \mathbf{0} \tag{3.1}$$

$$\xrightarrow{+} \quad \sum F_y = \mathbf{0} \tag{3.2}$$

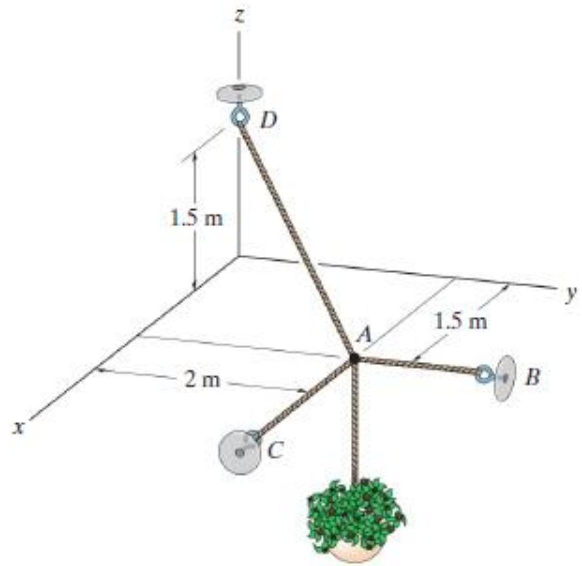
$$+\!\uparrow \quad \sum F_z = \mathbf{0}$$

These three equations state that the algebraic sum of the components of all the forces acting on the particle along each of the coordinate axes must be zero. Therefore the algebraic signs of these components should be selected according to chosen direction. If the arrow head of a vector is towards the selected direction, it has a positive magnitude whereas if the arrow head of a vector is opposite to the selected direction, it has a negative magnitude. Using them we can solve for at most three unknowns, generally represented as coordinate direction angles or magnitudes of forces shown on the particle's free-body diagram.

Three-dimensional force systems have same type of problems and solution procedure as in coplanar force systems.

Example 3.4

The three cables are used to support the 40-kg flowerpot. Determine the force developed in each cable for equilibrium.



Example 3.5

Each cord can sustain a maximum tension of 500 N. Determine the largest mass of pipe that can be supported.