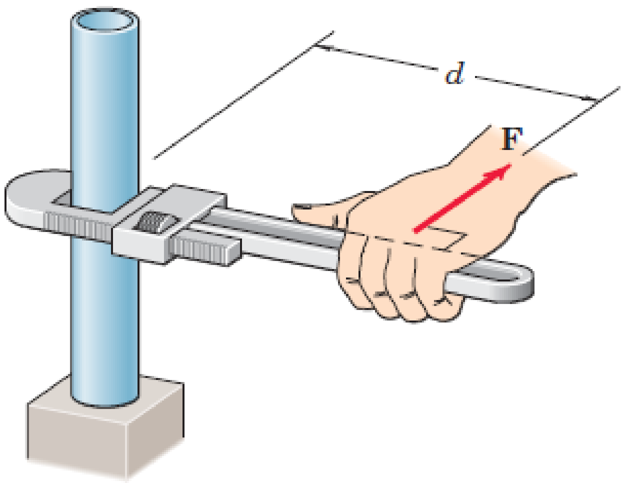
CHAPTER 4. FORCE SYSTEM RESULTANTS

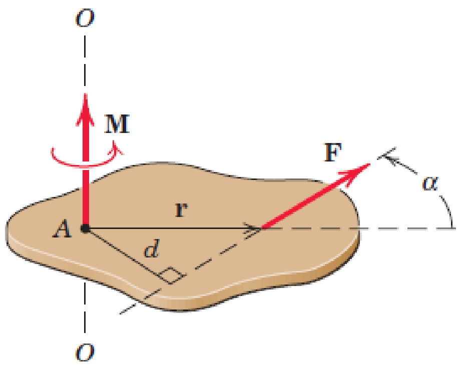
4.1. Moment of a Force (about a Point) – Scalar Formulation

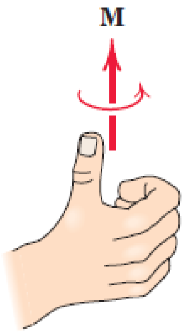
In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the moment M of the force. Moment is also referred to as torque.

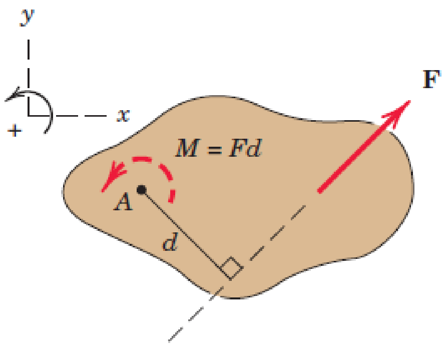


As a familiar example of the concept of moment, consider the pipe wrench of given figure. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude $\vec{F}$ of the force and the effective length d of the wrench handle.

Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.







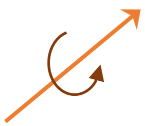
Magnitude. Left figure shows a two-dimensional body acted on by a force $\vec{F}$ in its plane. The magnitude of the moment or tendency of the force to rotate the body about the *point A* (*axis O-O*) perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

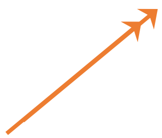
$$M_A = Fd \qquad (1)$$

NOTE: If the line of action of a force pass through a point A, the force does not create any moment about that point A. ($M_A = Fd = M_A = F0 = 0$)

Direction. The moment is a vector $\vec{M}$ perpendicular to the plane of the body. The sense of $\vec{M}$ depends on the direction in which F tends to rotate the body. The right-hand rule is used to identify this sense. We represent the moment of $\vec{F}$ about A (axis O-O) as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency.

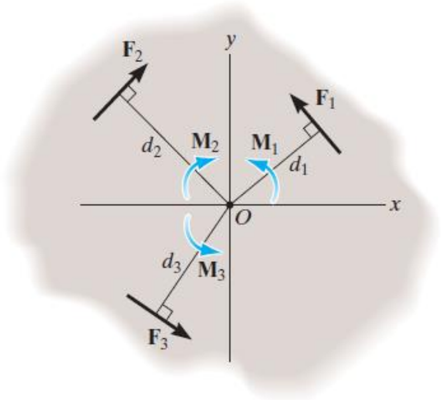
When dealing with forces which all act in a given plane, we customarily speak of the moment about a point. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Moment vector can be shown as,





Unit. The basic units of moment in SI units are newton-meters (**N.m** or Nm).

Sign Convention. As a convention, we will generally consider *positive moments* as counterclockwise since they are directed along the positive z axis (out of the page). Clockwise moments will be *negative*. Doing this, the directional sense of each moment can be represented by a plus or minus sign.



Resultant Moment. For two-dimensional problems, where all the forces lie within the same plane, the resultant moment $(\overrightarrow{M_R})_O$ about point O (the z axis) can be determined by finding the algebraic sum of the moments caused by all the forces in the system. Using sign convention given above, the directional sense of each moment can be represented by a plus or minus sign.

Using this sign convention, with a symbolic curl to define the positive direction, the resultant moment in figure is therefore

$$(M_R)_O = M_1 - M_2 + M_3 = F_1 d_1 - F_2 d_2 + F_3 d_3$$

If the numerical result of this sum is a positive scalar, $(M_R)_O$ will be a counterclockwise moment (out of the page); and if the result is negative, $(M_R)_O$ will be a clockwise moment (into the page).

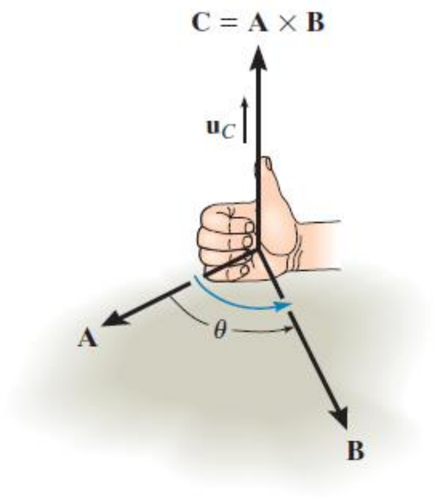
4.2. Cross Product

In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations. Before doing this, however, it is first necessary to expand our knowledge of vector algebra and introduce the cross-product method of vector multiplication, first used by Willard Gibbs in lectures given in the late 19th century.

The cross product of two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ yields the vector $\vec{\boldsymbol{C}}$, which is written

$$\vec{C} = \vec{A}\, x\, \vec{B}$$

and is read "C equals A cross B."



Magnitude. The magnitude of $\vec{\boldsymbol{C}}$ is defined as the product of the magnitudes of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ and the sine of the angle θ between their tails $(0 \leq \theta \leq 180)$. Thus,

$$C = AB\, sin\theta.$$

Direction. Vector $\vec{\boldsymbol{C}}$ has a direction that is perpendicular to the plane containing $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ such that $\vec{\boldsymbol{C}}$ is specified by the right-hand rule; i.e., curling the fingers of the right hand from vector $\vec{\boldsymbol{A}}$ (cross) to vector $\vec{\boldsymbol{B}}$, the thumb points in the direction of $\vec{\boldsymbol{C}}$, as shown in the left figure.

Knowing both the magnitude and direction of $\vec{\boldsymbol{C}}$, we can write

$$\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} x \vec{\boldsymbol{B}} = (AB\, sin\theta)\overrightarrow{\boldsymbol{u_C}} \qquad (2)$$

where the scalar $AB\ sin\theta$ defines the magnitude of $\vec{\boldsymbol{C}}$ and the unit vector $\overrightarrow{\boldsymbol{u_C}}$ defines the direction of $\vec{\boldsymbol{C}}$.

Laws of Operation.

i. The commutative law is not valid,

$$\vec{A}\ x\ \vec{B}\ \neq\ \vec{B}\ x\ \vec{A}.$$

Rather,

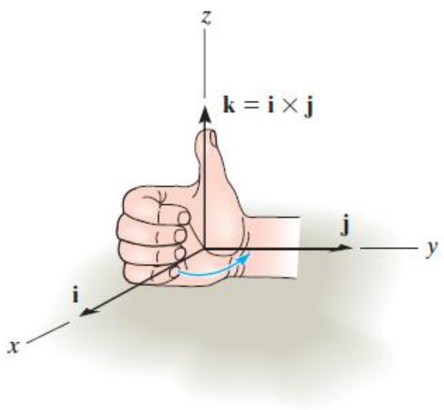
$$\vec{A}\ x\ \vec{B}\ = -\ \vec{B}\ x\ \vec{A}$$

ii. If the cross product is multiplied by a scalar a, it obeys the associative law,

$$a(\vec{A}\ x\ \vec{B})\ =\ (a\vec{A}\ )\ x\ \vec{B}\ =\ \vec{A}\ \ x\ (a\vec{B})\ =\ (\vec{A}\ x\ \vec{B})a$$

iii. The vector cross product also obeys the distributive law of addition,

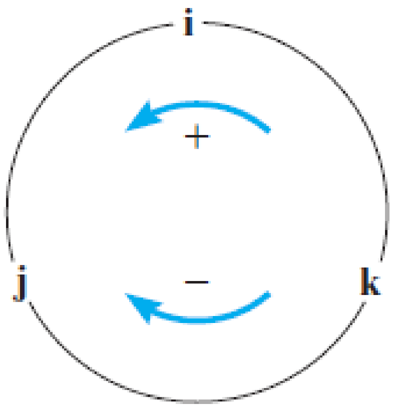
$$\vec{A}\ x\ (\vec{B} +\ \vec{D}) = (\vec{A}x\vec{B}) + (\vec{A}x\vec{D})$$



Cartesian Vector Formulation. Equation (2) may be used to find the cross product of any pair of Cartesian unit vectors. For example, to find $\vec{\imath}\ x\ \vec{\jmath}$, the magnitude of the resultant vector is $(i)(\ j)(sin\ 90)\ =\ (1)(1)(1)\ =\ 1$, and its direction is determined using the right-hand rule. As shown in the figure, the resultant vector points in the *+k* direction. Thus, $\vec{\imath}\ x\ \vec{\jmath}$ = (1) $\vec{k}$,. In a similar manner, following equations can be obtained.

$$\begin{array}{lll} \mathbf{i} \times \mathbf{j} = \mathbf{k} & \mathbf{i} \times \mathbf{k} = -\mathbf{j} & \mathbf{i} \times \mathbf{i} = \mathbf{0} \\ \mathbf{j} \times \mathbf{k} = \mathbf{i} & \mathbf{j} \times \mathbf{i} = -\mathbf{k} & \mathbf{j} \times \mathbf{j} = \mathbf{0} \\ \mathbf{k} \times \mathbf{i} = \mathbf{j} & \mathbf{k} \times \mathbf{j} = -\mathbf{i} & \mathbf{k} \times \mathbf{k} = \mathbf{0} \end{array}$$

Fig. 4.1. Cross product of unit vectors



These results should not be memorized; rather, it should be clearly understood how each is obtained by using the right-hand rule and the definition of the cross product. A simple scheme shown in the left figure is helpful for obtaining the same results when the need arises. If the circle is constructed as shown, then "crossing" two unit vectors in a counterclockwise fashion around the circle yields the positive third unit vector;

$$\vec{k} \, x \, \vec{\imath} = \vec{\jmath}$$

"Crossing" clockwise, a negative unit vector is obtained;

$$\vec{\imath} \, x \, \vec{k} = -\vec{\jmath}$$

Let us now consider the cross product of two general vectors $\vec{A}$ and $\vec{B}$ which are expressed in Cartesian vector form. We have;

$$\begin{aligned} \mathbf{A} \times \mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\ &= A_xB_x(\mathbf{i} \times \mathbf{i}) + A_xB_y(\mathbf{i} \times \mathbf{j}) + A_xB_z(\mathbf{i} \times \mathbf{k}) \\ &\quad + A_yB_x(\mathbf{j} \times \mathbf{i}) + A_yB_y(\mathbf{j} \times \mathbf{j}) + A_yB_z(\mathbf{j} \times \mathbf{k}) \\ &\quad + A_zB_x(\mathbf{k} \times \mathbf{i}) + A_zB_y(\mathbf{k} \times \mathbf{j}) + A_zB_z(\mathbf{k} \times \mathbf{k}) \end{aligned}$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

This equation may also be written in a more compact determinant form as

$$\vec{A}x\vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Thus, to find the cross product of any two Cartesian vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$, it is necessary to expand a determinant whose first row of elements consists of the unit vectors $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$, and whose second and third rows represent the *x, y, z* components of the two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$, respectively.

The determinant process is summarized in the figure below.

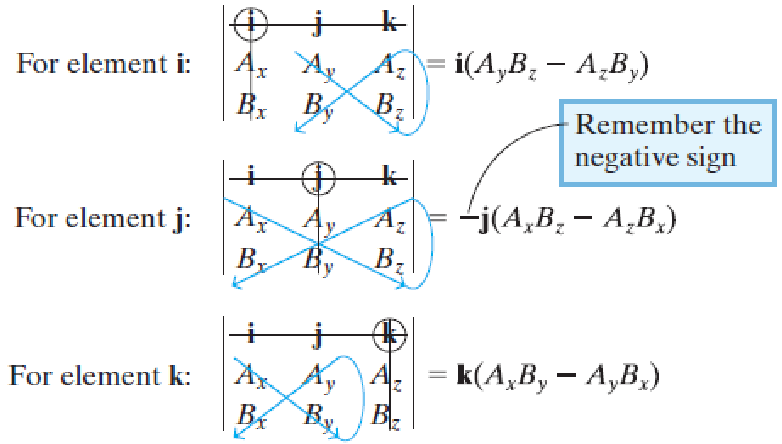
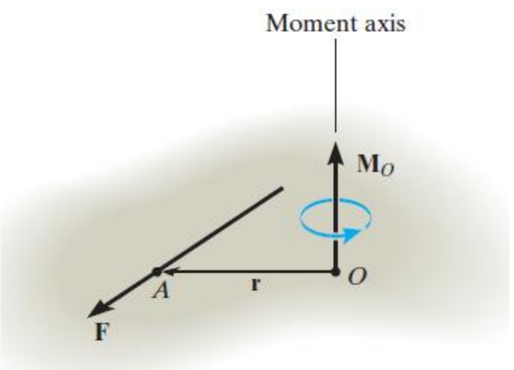
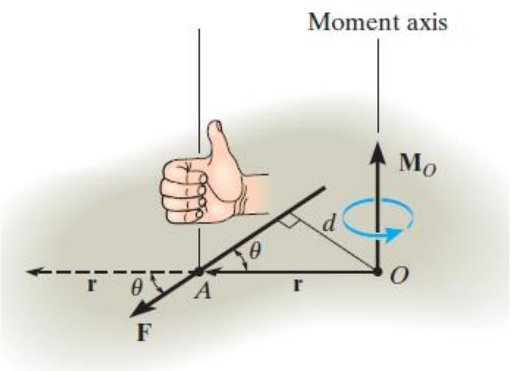


Fig. 4.2. Summation of a determinant process

4.3. Moment of a Force (about a Point) — Vector Formulation



(a)



(b)

The moment of a force $\vec{F}$ about point O, or actually about the moment axis passing through O and perpendicular to the plane containing O and $\vec{F}$, can be expressed using the vector cross product, namely,

$$\vec{M}_O = \vec{r} x \vec{F}$$

Here $\vec{r}$ represents a position vector directed from O to any point on the line of action of $\vec{F}$.

NOTE: Since the cross product does not obey the commutative law ($\vec{r} x \vec{F} \neq \vec{F} x \vec{r}$), the order of $\vec{r} x \vec{F}$ must be maintained to produce the correct sense of direction for $\vec{M}_O$.

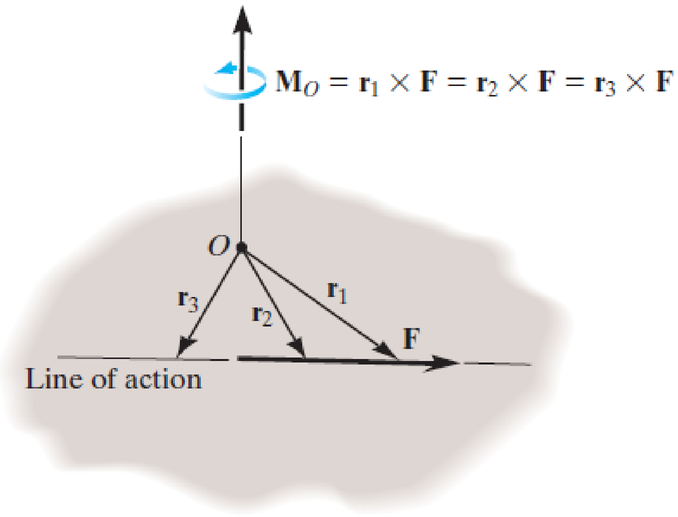
Magnitude. The magnitude of the cross product is defined as

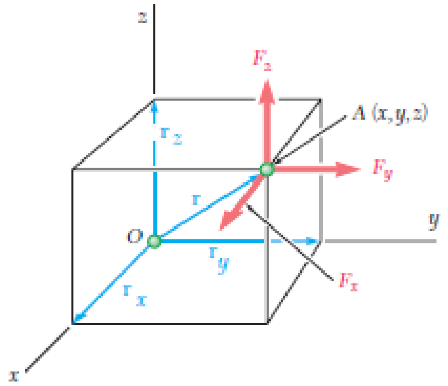
$$M_O = rFsin\theta$$

where the angle θ is measured between the tails of $\vec{r}$ and $\vec{F}$. To establish this angle, $\vec{r}$ must be treated as a sliding vector so that θ can be constructed properly.

$$M_O = rFsin\theta = F(rsin\theta) = Fd$$

Direction. The direction and sense of $\vec{M}_O$ are determined by the right-hand rule as it applies to the cross product.





Principle of Transmissibility. Force is a floating (sliding) vector and can move freely on its line of action. In other words, we can use any position vector $\vec{r}$ measured from point O to any point on the line of action of the force $\vec{F}$.

Cartesian Vector Formulation. If position vector $\vec{r}$ (from O to the any point on the line of action of $\vec{F}$) and force $\vec{F}$ are expressed as Cartesian vectors, the moment about a point O, $\vec{M}_O$, can be expressed as follows.

$$\vec{M}_O = \vec{r} x \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{M}_O = (r_y F_z - r_z F_y)\,\vec{i} - (r_x F_z - r_z F_x)\,\vec{j} + (r_x F_y - r_y F_x)\,\vec{k}$$

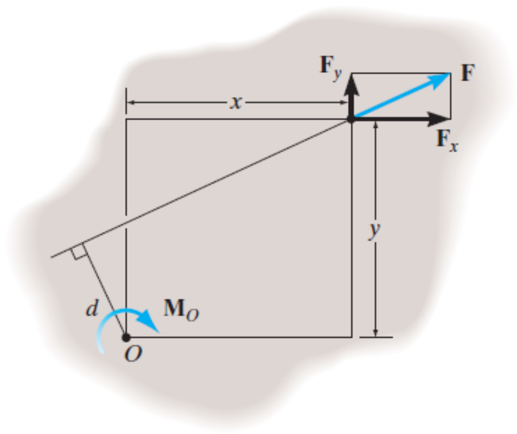
Resultant Moment of a System of Forces. If a body is acted upon by a system of forces, the resultant moment of the forces about point O can be determined by vector addition of the moment of each force. This resultant can be written symbolically as

$$\vec{M}_O = \sum (\vec{r} x \vec{F})$$

4.4. Varignon's Theorem (Principle of Moments)

It states that the moment of a force about a point is equal to the sum of the moments of the components of the force about the point. This theorem can be proven easily using the vector cross product since the cross product obeys the distributive law.

$$\vec{\boldsymbol{M}}_{\boldsymbol{O}} = \vec{\boldsymbol{r}} x \vec{\boldsymbol{F}} = \vec{\boldsymbol{r}} x\left(\vec{\boldsymbol{F}}_x + \vec{\boldsymbol{F}}_y\right) = \vec{\boldsymbol{r}} x \vec{\boldsymbol{F}}_x + \vec{\boldsymbol{r}} x \vec{\boldsymbol{F}}_y = (\boldsymbol{x}\vec{\boldsymbol{\imath}} + \boldsymbol{y}\vec{\boldsymbol{\jmath}})\boldsymbol{x}(\boldsymbol{F}_x\vec{\boldsymbol{\imath}}) + (\boldsymbol{x}\vec{\boldsymbol{\imath}} + \boldsymbol{y}\vec{\boldsymbol{\jmath}})\boldsymbol{x}(\boldsymbol{F}_y\boldsymbol{y}\vec{\boldsymbol{\jmath}}) = -\boldsymbol{F}_x\boldsymbol{y}\vec{\boldsymbol{k}} + \boldsymbol{F}_y\boldsymbol{x}\vec{\boldsymbol{k}}$$



For two-dimensional problems, we can use the Varignon's theorem by resolving the force into its rectangular components and then determine the moment using a scalar analysis. Thus,

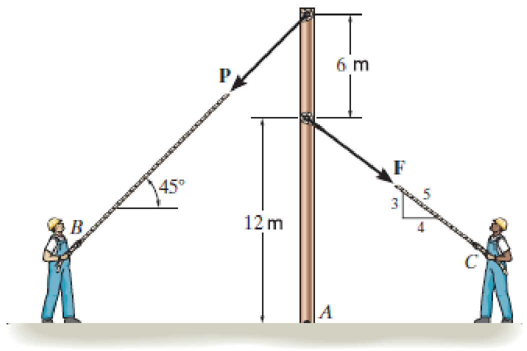
$$M_O = -F_x\, y + F_y\, x$$

This method is generally easier than finding the same moment using

$$M_O = Fd$$

Example 1.

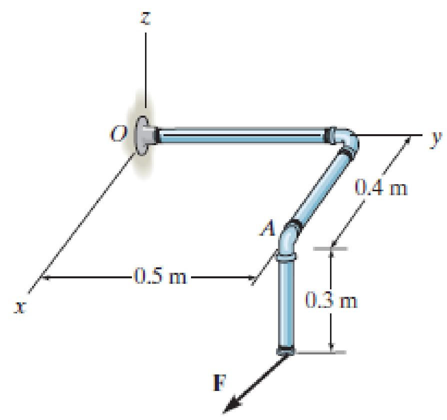
If the man *at B* exerts a force of P = 300N on his rope, determine the magnitude of the force $\vec{F}$ the man at *C* must exert to prevent the pole from rotating, i.e., so the resultant moment about *A* of both forces is zero.



Solution:

Example 2.

Determine the coordinate direction angles α, β, γ of force $\vec{F}$, so that the moment of $\vec{F}$ about O is zero.

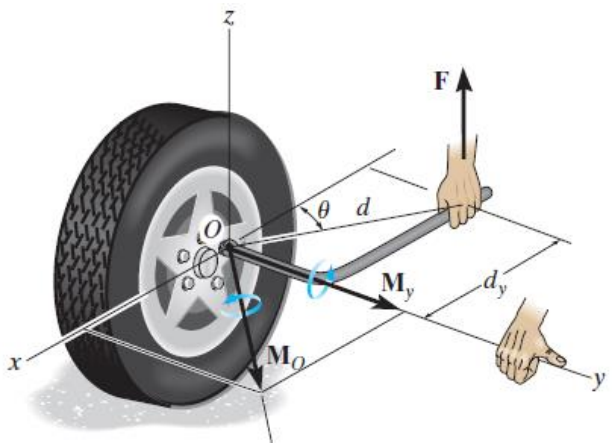


Solution:

4.5. Moment of a Force about a Specified Axis

Sometimes, the moment produced by a force about a specified axis must be determined. To determine this component, we can use either a scalar or vector analysis.

Scalar Analysis



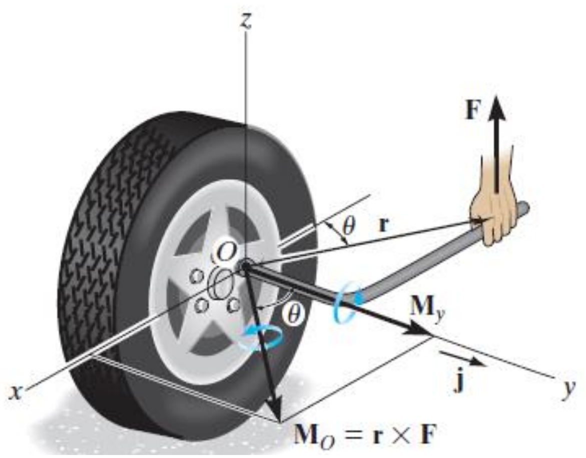
Similar to moment of a force about a point, for any axis *a*, the magnitude of the moment is equal to multiplication of magnitude of the force and the perpendicular distance from the axis *a* to the line of action of the force. The direction of moment can be found by right-hand rule. When we curl our fingers in the direction of the rotational tendency of the moment around axis *a*, if our thumb shows the direction of the *a* axis, then moment is accepted as positive, otherwise, moment is negative.

$$M_a = Fd_a$$

For the given figure, the moment produced by $\vec{F}$ about *y* axis is equal to,

$$M_y = Fd_y$$

Vector Analysis



To find the moment of a force $\vec{F}$ about *a* axis using a vector analysis,

i. we must first determine the moment of $\vec{F}$ about any point A on the *a* axis by applying

$$\vec{M}_A = \vec{r} x \vec{F}$$

ii. The component M_a along the a axis is the projection of $\vec{M}_A$ onto the *a* axis.

It can be found using the dot product discussed in Chapter 2, so that

$$M_a = \vec{u}_a \cdot \vec{M}_A = \vec{u}_a \cdot (\vec{r} x \vec{F}) \qquad (3)$$

where, $\vec{u}_a$ is the unit vector in the direction of *a* axis.

For the given figure, the moment of $\vec{F}$ about *y* axis can be found as follows.

i. we must first determine the moment of $\vec{F}$ about any point such as *O* on the *y* axis

$$\vec{M}_O = \vec{r} x \vec{F}$$

ii. The component M_y along the y axis is the projection of $\vec{M}_A$ onto the *y* axis.

$$M_y = \vec{M}_O \cdot \vec{j}$$

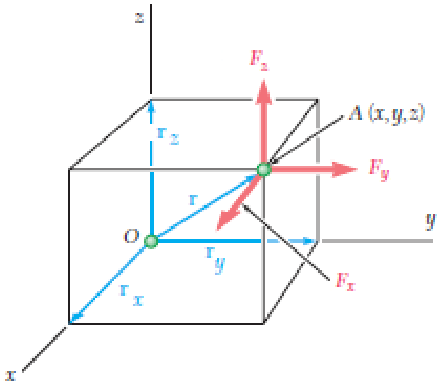
Eq. (3) (right side of the equation) is referred to as the scalar triple product. If the vectors are written in Cartesian form, we have

$$M_{\boldsymbol{a}} = \vec{u}_a \cdot (\vec{r} x \vec{F}) = \left[u_{a_x}\vec{\boldsymbol{\imath}} + u_{a_y}\vec{\boldsymbol{\jmath}} + u_{a_z}\vec{\boldsymbol{k}}\right] \begin{vmatrix} \vec{\boldsymbol{\imath}} & \vec{\boldsymbol{\jmath}} & \vec{\boldsymbol{k}} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x) =$$

$$M_{\boldsymbol{a}} = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \tag{4}$$

When $M_{\boldsymbol{a}}$ is evaluated from Eq. (4), it will yield a positive or negative scalar. The sign of this scalar indicates the sense of direction of $M_{\boldsymbol{a}}$ along the *a* axis. If it is positive, then $M_{\boldsymbol{a}}$ will have the same sense as $u_{\boldsymbol{a}}$, whereas if it is negative, then $M_{\boldsymbol{a}}$ will act opposite to $u_{\boldsymbol{a}}$. After $M_{\boldsymbol{a}}$ is determined, $\vec{\boldsymbol{M}}_{\boldsymbol{a}}$ can be easily found by,

$$\vec{\boldsymbol{M}}_{\boldsymbol{a}} = M_{\boldsymbol{a}}\vec{u}_a$$



$$\vec{M}_O = (r_yF_z - r_zF_y)\,\vec{\imath} + (-(r_xF_z - r_zF_x))\,\vec{\jmath} + (r_xF_y - r_yF_x)\,\vec{k}$$

$$= M_{0x}\,\vec{\imath} + M_{0y}\,\vec{\jmath} + M_{0z}\,\vec{k}$$

i) If the line of action of a force is parallel to the axis, no moment effect occurs about that axis. In other words, moment of the force about that axis is 0.

ii) If the line of action of a force pass through an axis, no moment effect occurs about that axis.

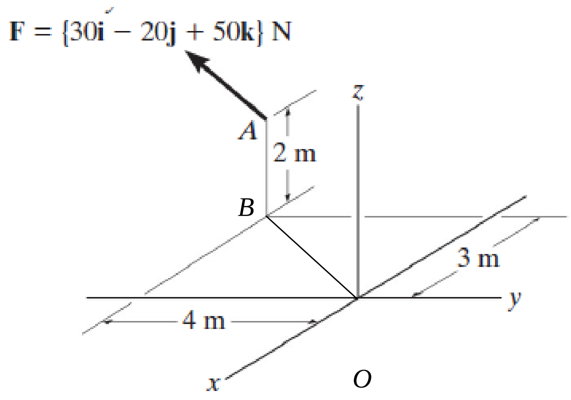
Resultant Moment of a System of Forces. If a body is acted upon by a system of forces, the resultant moment of the forces about *a* axis can be determined by addition of the moment of each force.

Example 3.

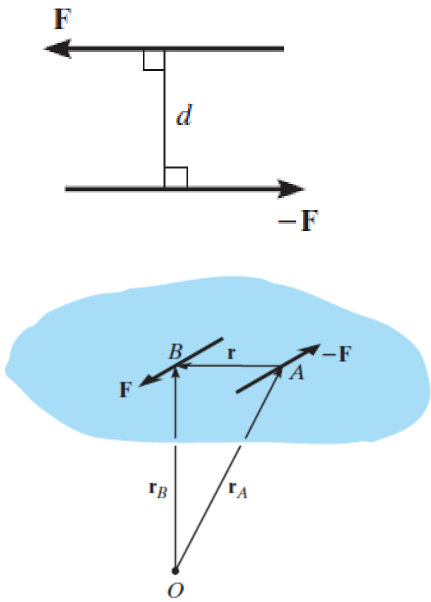
Determine the magnitude of the moment of the force

a) about the *y* axis.

b) about OB axis.



4.6. Moment of a Couple



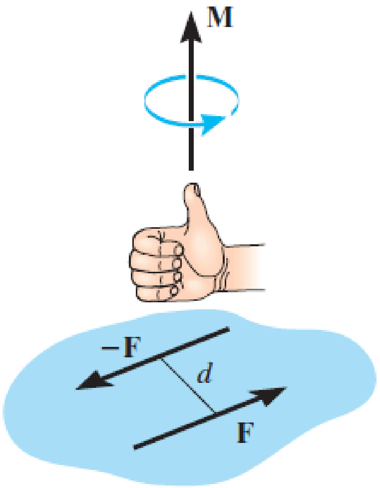
A **couple** is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d, Fig. 4–25. Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

The moment produced by a couple is called a **couple moment**. We can determine its value by finding the sum of the moments of both couple forces about any arbitrary point.

$\vec{\boldsymbol{M}} = \vec{r}_B x\vec{\boldsymbol{F}} + \vec{r}_A x(-\vec{\boldsymbol{F}}) = (\vec{r}_B - \vec{r}_A)\boldsymbol{x}\vec{\boldsymbol{F}} = \vec{\boldsymbol{r}}\boldsymbol{x}\vec{\boldsymbol{F}}$ **(vector formulation)**

This result indicates that a couple moment is a free vector, *i.e.*, it can act at any point since $\vec{\boldsymbol{M}}$ depends only upon the position vector $\vec{\boldsymbol{r}}$ directed between the forces, not $\vec{\boldsymbol{r}}_{\boldsymbol{A}}$ or $\vec{\boldsymbol{r}}_{\boldsymbol{B}}$.

Scalar Formulation



The moment of a couple, $\vec{\boldsymbol{M}}$, given in the left figure, is defined as having a magnitude of

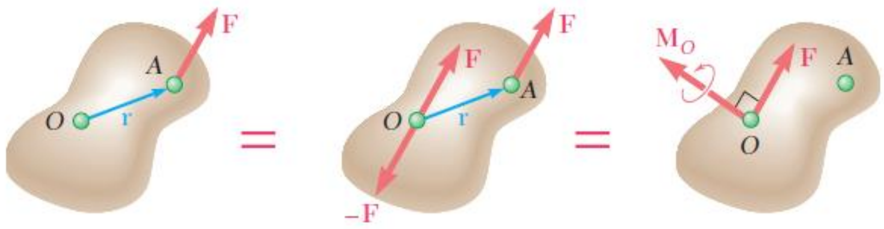
$$M = Fd$$

where F is the magnitude of one of the forces and d is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right-hand rule, where the thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces. In all cases, $\vec{\boldsymbol{M}}$ will act perpendicular to the plane containing these forces.

Equivalent Couples. If two couples produce a moment with the same magnitude and direction, then these two couples are equivalent.

Resultant Couple Moment. Since couple moments are vectors, their resultant can be determined by vector addition.

4.7. Simplification of a Force and Couple System

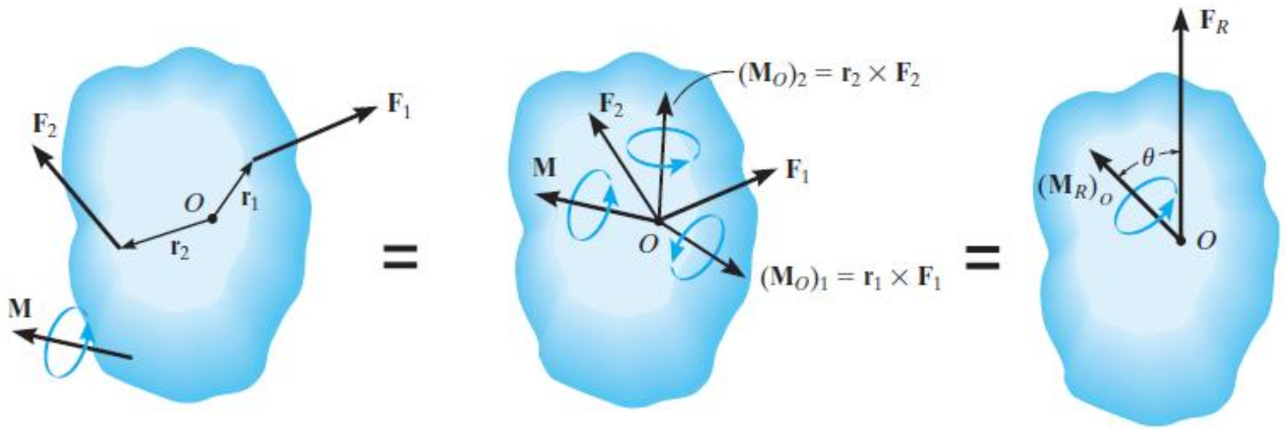


Consider a force $\vec{F}$ acting on a rigid body at a point A defined by the position vector $\vec{r}$ (Fig. 3.39 a). Suppose that for some reason we would rather have the force act at point O. While we can move $\vec{F}$ along its line of action (principle of transmissibility), we cannot move it to a point O which does not lie on the original line of action without modifying the action of $\vec{F}$ on the rigid body.

We can, however, attach two forces at point O , one equal to $\vec{F}$ and the other equal to $-\vec{F}$, without modifying the action of the original force on the rigid body. As a result of this transformation, a force $\vec{F}$ is now applied at O ; the other two forces form a couple of moment $\vec{\boldsymbol{M}} = \vec{\boldsymbol{r}}\boldsymbol{x}\vec{\boldsymbol{F}}$. Thus, any force $\vec{F}$ acting on a rigid body can be moved to an arbitrary point O provided that a couple is added whose moment is equal to the moment of $\vec{F}$ about O.

Sometimes it is convenient to reduce a system of forces and couple moments acting on a body to a simpler form by replacing it with an equivalent system, consisting of a single resultant force acting at a specific point and a resultant couple moment. A system is equivalent if the external effects it produces on a body are the same as those caused by the original force and couple moment system.

System of Forces and Couple Moments. Using the above method, a system of several forces and couple moments acting on a body can be reduced to an equivalent single resultant force acting at a point O and a resultant couple moment.



NOTES:

i. Force is a sliding vector, since it will create the same external effects on a body when it is applied at any point P along its line of action. This is called the principle of transmissibility.

ii. A couple moment is a free vector since it will create the same external effects on a body when it is applied at any point P on the body.

iii. When a force is moved to another point P that is not on its line of action, it will create the same external effects on the body if a couple moment is also applied to the body. The couple moment is determined by taking the moment of the force about point P.

Procedure for Analysis:

Assume that we want reduce a system of several forces and couple moments acting on a body to an equivalent system acting about a point O.

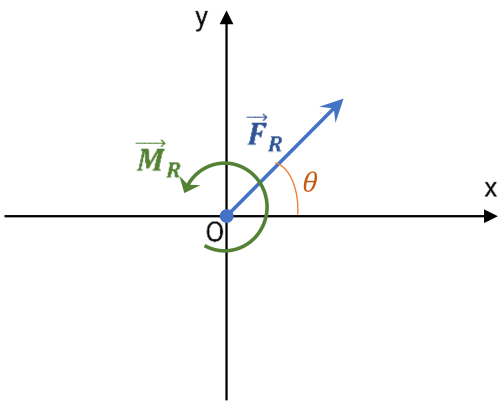
i. If not given in the problem, establish the x and y axis in any suitable orientation. Choosing a good orientation makes the solution simpler.

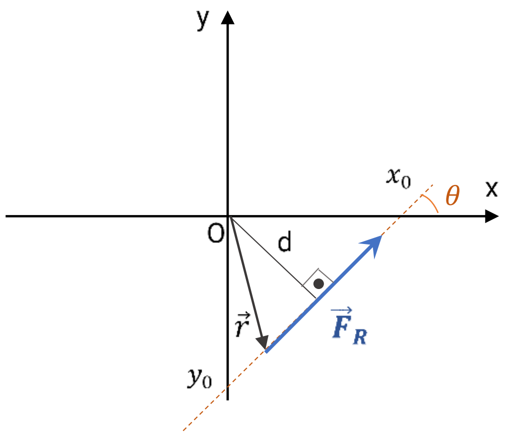
ii. If the force system is coplanar, resolve each force into its x and y components. If a component is directed along the positive x or y axis, it represents a positive scalar; whereas if it is directed along the negative x or y axis, it is a negative scalar. In three dimensions, represent each force as a Cartesian vector before summing the forces. In other words, the forces acting on the system will be added as if all of them acting at the same point, O.

iii. When determining the moments of a coplanar force system about point O, it is generally advantageous to use Varignon's theorem, i.e., determine the moments of the components of each force, rather than the moment of the force itself. In three dimensions use the vector cross product or scalar formulation to determine the moment of each force about point O. Do not forget to add couple moments (if exist) acting on the system.

4.8. Further Simplification of a Force and Couple System

After reducing the system of several forces and couple moments into the equivalent one resultant force and one resultant moment system, the resultant moment can be replaced by moving the resultant force away from point O such that the resultant force produces the same moment resultant moment about point O. Perpendicular distance from point O to new line of action of $\vec{\boldsymbol{F}}_{\boldsymbol{R}}$ can be found by scalar moment equation, $M_R = F_R d$. In order to find the equation of the new line of action of $\vec{\boldsymbol{F}}_{\boldsymbol{R}}$, moment equation in vector form can be used.





$$\overrightarrow{M}_R = \vec{r} x \overrightarrow{F}_R$$

$$\vec{r} = r_x \vec{i} + r_y \vec{j}, \quad \overrightarrow{F}_R = F_x \vec{i} + F_y \vec{j}$$

$$\overrightarrow{M}_R = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & 0 \\ F_x & F_y & 0 \end{vmatrix} = \vec{k}\left(r_x F_y - r_y F_x\right)$$

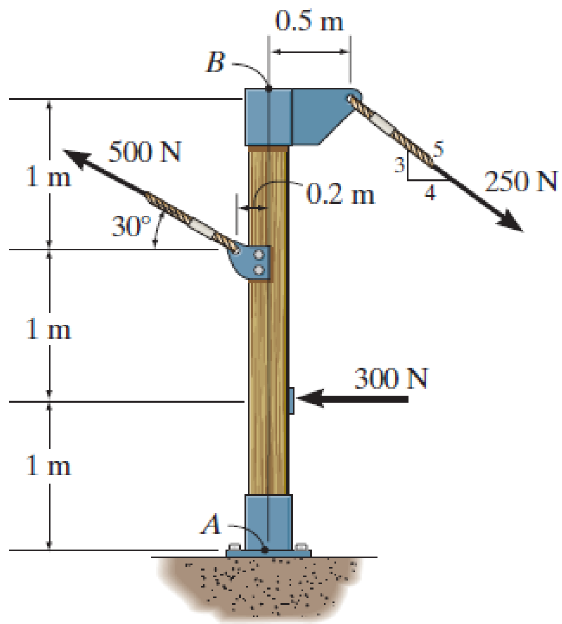
$$M_R = r_x F_y - r_y F_x \quad (5)$$

Eq. (5) (substitute $r_x \rightarrow x, r_Y \rightarrow y$) gives the line equation of new line of action. From Eq (5), x_0 and y_o which are the intersect points of new line of action can be calculated (or using trigonometry).

$$x_0 = \frac{M_R}{F_y}, \quad y_0 = -\frac{M_R}{F_x} \text{ (actual values with signs) or } \quad x_0 = \frac{d}{\sin\theta}, \quad y_0 = \frac{d}{\cos\theta} \text{ (absolute values, signs should be added)}$$

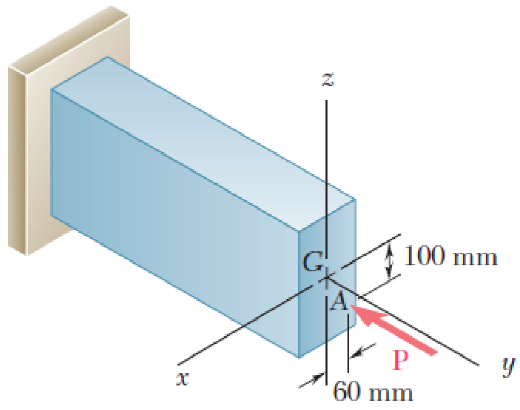
Example 4.

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.



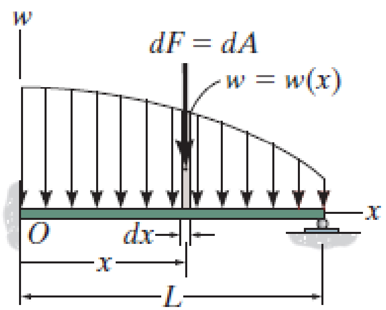
Example 5.

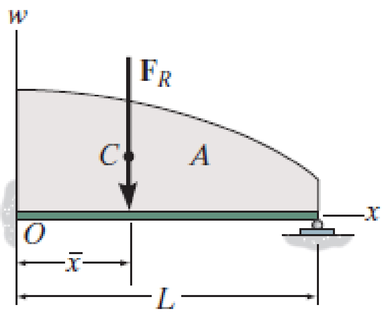
An eccentric, compressive 1220-N force $\vec{P}$ is applied to the end of a cantilever beam. Replace $\vec{P}$ with an equivalent force-couple system at G .



4.9. Reduction of a Simple Distributed Loading

Sometimes, a body may be subjected to a loading that is distributed over its surface. For example, the pressure of the wind on the face of a sign, the pressure of water within a tank, or the weight of sand on the floor of a storage container, are all distributed loadings. It is measured using $\frac{N}{m}$ (2D) or $\frac{N}{m^2}$ (3D) in SI.





Using the methods of force system simplification methods, we can replace this coplanar parallel force system with a single equivalent resultant force $\vec{F}_R$ acting at a specific location on the beam.

Magnitude of Resultant Force

The magnitude of $\vec{F}_R$ is equivalent to the sum of all the forces in the system. In this case integration must be used since there is an infinite number of parallel forces $d\vec{F}$ acting on the beam.

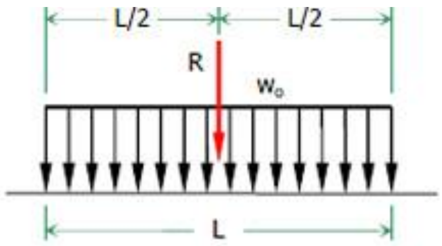
$$+\downarrow \quad \vec{F}_R = \sum \vec{F} = \int_0^L w(x)dx = \int_A dA = A$$

Location of Resultant Force

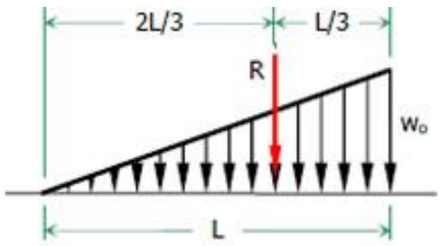
Since both system are equilant to each other, both has to have same moments. The location x of the line of action of $\vec{F}_R$ can be determined by equating the moments of the force resultant and the parallel force distribution about point O (the y axis).

$$\circlearrowleft + \quad (M_R)_O = -F_R\bar{x} = -\int_0^L w(x)xdx \qquad \bar{x} = \frac{\int_0^L w(x)xdx}{\int_0^L w(x)dx}$$

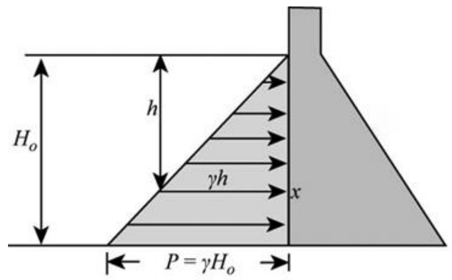
Some Simple Distributed Loadings



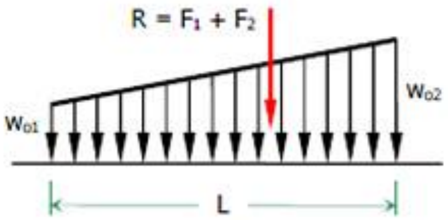
Rectangular Load



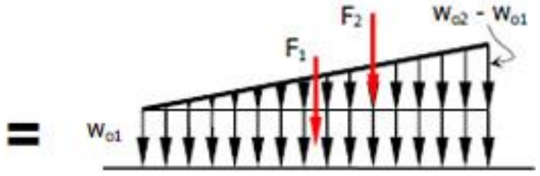
Triangular Load



γ is the unit weight $(\frac{N}{m^3})$ of the water.



Trapezoidal Load



Resolution of trapezoidal load into rectangular and triangular loads

Example 6.

Determine the resultant force and specify where it acts on the beam measured from A.