CHAPTER 1. INTRODUCTION TO STATICS

1.1. Mechanics

Mechanics is the physical science which deals with the effects of forces on objects. It is one of the oldest of the physical sciences. It aims to clarify physical phenomena and enlighten engineering applications by predicting their consequences. Mechanics is the foundation of most engineering sciences as in the civil engineering and is an indispensable prerequisite to their study. It is based on only a few laws of nature such as Newton's three laws. These laws are statements based on numerous observations and regarded as being known from experience. It should be noted that mechanics as a science are rigorously expressed by mathematics, and thus mathematics plays an important role in the application of these laws to the solution of practical problems.

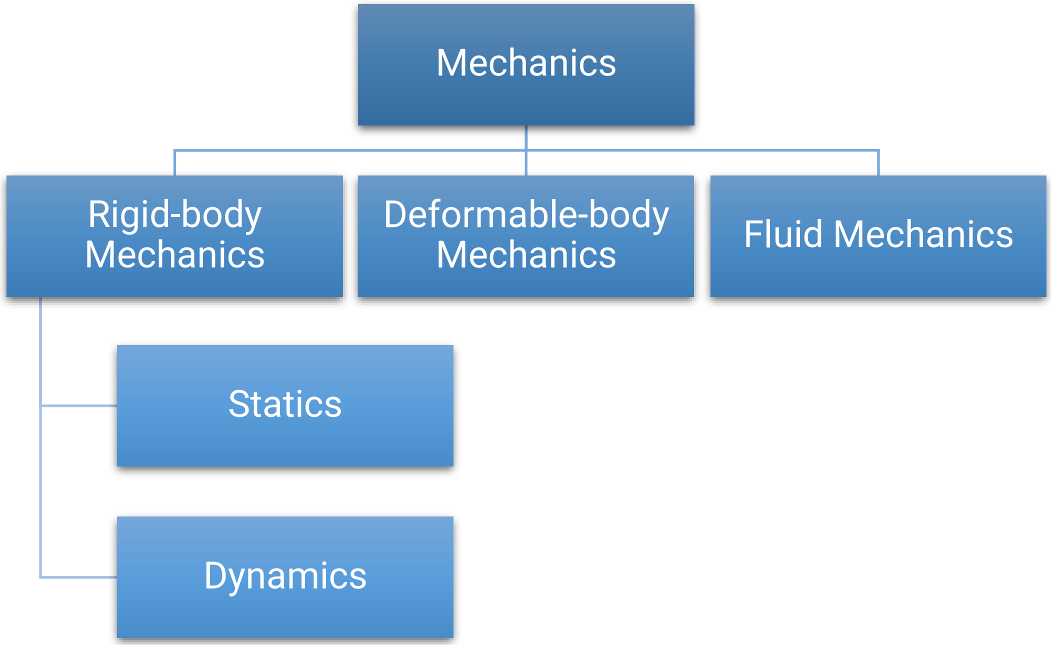


Fig. 1.1. The branches of mechanics

In general, Mechanics can be subdivided into three branches: **rigid-body mechanics**, **deformable-body mechanics**, and **fluid mechanics**. Rigid-body mechanics is divided into two areas: **statics** and **dynamics**. Statics deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity; whereas dynamics is concerned with the accelerated motion of bodies. In this course, we will study Statics since it is a basic requirement for the study of Dynamics, the mechanics of deformable bodies and the mechanics of fluids.

The subject of statics developed very early in history because its principles can be formulated simply from measurements of geometry and force. The earliest recorded writings are those of Archimedes (287–212 B.C.) on the principle of the lever. Substantial progress came later with the formulation of the laws of vector combination of forces by Stevinus (1548–1620), who also formulated most of the principles of statics.

1.2. Basic Concepts

Before we begin our study of mechanics, it is important to understand the meaning of certain basic concepts (terms) which will be used throughout the mechanic courses.

Length is used to locate the position of a point in space and describe the size of a physical system. Once a standard unit of length is defined, one can then use it to define distances and geometric properties of a body as multiples of this unit.

Time is conceived as a succession of events. Although the principles of statics are time independent, this quantity plays an important role in the study of dynamics.

Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its magnitude, by the direction of its action, and by its point of application. Thus force is a vector quantity, and it will be discussed in detail in Chapter 2.

In any physical system, if the main elements or parameters are taken into account and trivial ones are ignored, this process is called **idealization**. Idealizations are frequently used in mechanics in order to simplify application of the theory. Here we will consider three important idealizations.

A **particle** is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point. We often choose a particle as a differential element of a body. We may treat a body as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form since the geometry of the body will not be involved in the analysis of the problem.

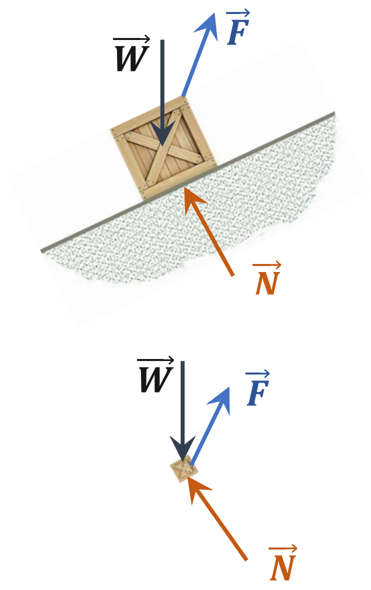


Fig. 1.2. A particle example

A **rigid body** can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load. This model is important because the body's shape does not change when a load is applied (no matter how big the force is), and so we do not have to consider the type of material from which the body is made. Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium. Determination of the internal deformations belongs to the study of the mechanics of deformable bodies, in other words, Mechanics of Materials courses at the second year.

A **concentrated force** represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body.

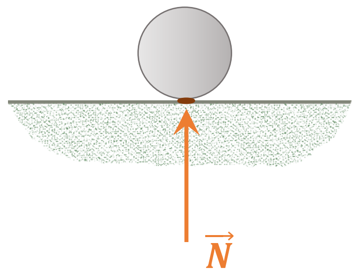


Fig. 1.3. Concentrated Force

1.3. System of Units

In mechanics, there are four basic quantities namely, length, time, mass and force. They are not all independent from one another; in fact, they are related by Newton's second law of motion ($\vec{F}=m\vec{a}$). Because of this, the units used to measure these quantities cannot all be selected arbitrarily. The equality $\vec{F}=m\vec{a}$ is maintained only if three of the four units, called ***base units,*** are defined (chosen) and the fourth unit is then derived from the equation. The International System of Units, abbreviated SI, is accepted throughout the world, and is a modern version of the metric system (MKS). By international agreement, SI units will in time replace other systems; and SI units will be used in this course, or generally in all mechanic courses. The four basic quantities and their units and symbols in different system of units are given in the Table 1.1. It should be noted that, it is possible to pass from one system of unit to another.

Table 1.1. Units and symbols in different system of units

System of Unit		Length	Mass	Time	Force
SI	unit	meter	kilogram	second	newton*
	symbol	*m*	*kg*	*s*	$N\ \left(kg\,\frac{m}{s^2}\right)$
CGS	unit	centimeter	gram	second	din*
	symbol	*cm*	*g*	*s*	$din\ \left(g\,\frac{cm}{s^2}\right)$
FPS (US Customary)	unit	foot	slug*	second	pound
	symbol	*ft*	$slug\left(\frac{lb\cdot s^2}{ft}\right)$	*s*	*lb*

* derived unit

In SI units, **1 newton** is equal to a force (derived from $\vec{F}=m\vec{a}$) required to give 1 kilogram of mass an acceleration of 1 m/s^2.

Table 1.2. Unit Conversion Factors

Quantity	SI	FPS
Length	1 m	~3.2808 ft
Mass	1 kg	~0.0685 slug
Force	1 N	~0.2248 lb

Example 1.1

Convert the gravitational acceleration $g=9.81\ m/s^2$ (SI) to ft/s^2 (FPS).

Solution:

m→ft

$$g = 9.81\frac{m}{s^2} = 9.81\frac{3.28ft}{s^2} = 32.18\frac{ft}{s^2}$$

Example 1.2

How many din is 1 newton?

Solution:

$$N\left(kg\frac{m}{s^2}\right) \rightarrow din\left(g\frac{cm}{s^2}\right)$$

$$1N = 1kg\frac{m}{s^2} = 1(1000g)\frac{(100cm)}{s^2} = 100000g\frac{cm}{s^2} = 100000din = 10^5 din$$

Prefixes: When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix. Some of the prefixes used in the SI system are shown in Table 1.3.

Table 1.3. Prefixes

Multiply (10^x)	Prefix	SI Symbol	
10^9	giga	*G*	*Large Numbers (Multiplies)*
10^6	mega	*M*	
10^3	kilo	*k**	
10^{-3}	milli	*m**	*Small Numbers (Submultiples)*
10^{-6}	macro	*μ*	
10^{-9}	nano	*n*	

* mostly used in Statics

Removing Prefix:

$10\ \boldsymbol{k}N = 10 * 10^3\ N = 10000\ N$ (decimal point moves three places to right)

$14\ \boldsymbol{m}m = 14 * 10^{-3}\ m = 0.014\ m$ (decimal point moves three places to left)

Applying Prefix:

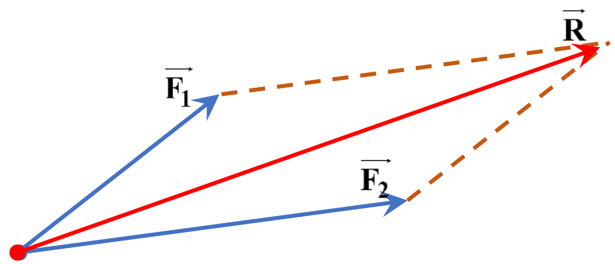
(g→**k**g**)** $132000\ g = 132 * 10^3\ g = 132\ kg$

(s→**m**s**)** $0.123\ s = 123 * 10^{-3}$ s = 123 ms

1.4. Fundamental Principles of Statics

The study of Statics rests on six fundamental principles based on experimental evidence.

i. The Parallelogram Law for the Addition of Forces. This states that two forces acting on a particle may be replaced by a single force, called their resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces.



$$\vec{R}=\vec{F}_1+\vec{F}_2$$

Fig. 1.4. Force Resultant

ii. The Principle of Transmissibility. This states that the conditions of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action. In other words, the principle of transmissibility states that the point of application of a force can be moved anywhere along its line of action without changing conditions of equilibrium or of motion of a rigid body (in Statics).

Fig. 1.5. The principle of transmissibility

iii-v. Newton's Three Fundamental Laws. Formulated by Sir Isaac Newton in the latter part of the seventeenth century.

iii. First Law. If the resultant force acting on a particle is zero (balanced force), the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

iv. Second Law. The resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force. If $\vec{F}$ is applied to a particle of mass *m*, this law may be expressed mathematically as $\vec{\mathbf{F}}=m\vec{\mathbf{a}}$.

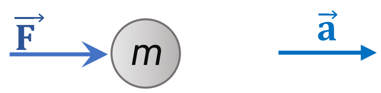


Fig. 1.6. Accelerated motion

Newton's first law contains the principle of the equilibrium of forces, which is the main topic of concern in Statics. This law is actually a consequence of the second law, since there is no acceleration when the force is zero, and the particle either is at rest or is moving with a uniform velocity.

v. Third Law. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line). It states that forces (between interacting bodies) always occur in pairs of equal and opposite forces.

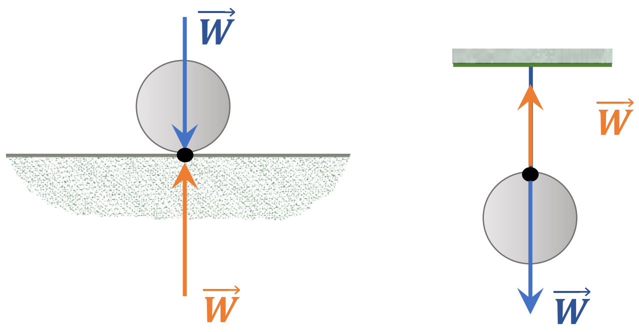


Fig. 1.7. Action reaction forces

For example, the downward force exerted on the ground by a ball is accompanied by an upward force of equal magnitude exerted on the ball by the ground. Lack of careful attention to this basic law is the cause of frequent error in Statics (generally in Mechanics). Because, it is necessary first of all to isolate the body under consideration and obtain free body diagram of the body.

vi. Newton's Law of Gravitational Attraction. Newton postulated a law governing the gravitational attraction between any two particles. This law states that two particles of mass M and m are mutually attracted with equal and opposite forces $\vec{\mathbf{F}}$ and $-\vec{\mathbf{F}}$ of magnitude F given by the formula

$$F = G\frac{Mm}{r^2} \qquad (1)$$

where r is the distance between the two particles and G is the universal constant called the **constant of gravitation** and equals $G=66.73*10^{-12}\ m^3/(kg.s^2)$**.**

The force $\vec{\mathbf{F}}$ exerted by the earth on the particle is then defined as the ***weight*** $\overrightarrow{\mathbf{W}}$ of the particle. Taking *M* equal to the mass of the earth, *m* equal to the mass of the particle, and *r* equal to the radius *R* of the earth, the weight (magnitude) of a particle can be expressed as follows.

$$W = G\frac{M}{R^2}m \tag{2}$$

Letting $g = G\frac{M}{R^2}$ yields

$$W = mg \tag{3}$$

Since *g* ,named as **gravitational acceleration,** depends on R, then the weight of a body is not an absolute quantity. Instead, its magnitude is determined from where the measurement was made. However, as long as the point actually remains on the surface (near surface) of the earth, it is sufficiently accurate in most engineering computations to assume that *g* equals **9.81** m/s^2. It must be noted that weight is a force (vector) and it always points to the center of the earth (or practically to downward in most of the problems in Mechanics).