MEASURING INSTRUMENTS

The structure of MOVING COIL METERS(Electromechanical Indicating Meters)

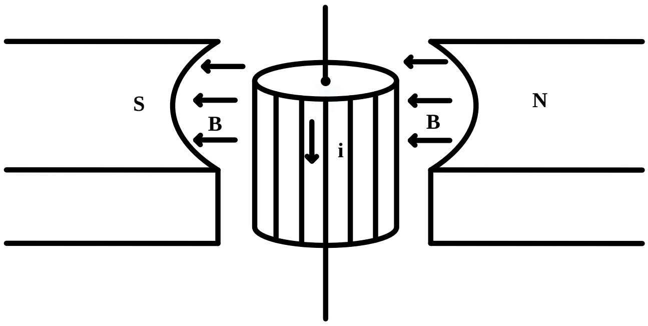


Figure. A simplified structure of a moving coil meter

These instruments consist of a coil which has a surface area S and N-turn, put in the air gap of a magnetic circuit fed by a permanent magnet with the strength of B, radial uniform magnetic induction field.

When a current i passes through the coil static balance equation of the movable part can be written as

$$c.\alpha = B.S.N.i \qquad c.\alpha = G.i$$

If the equivalent resistance of the movable coil itself and circulating the circuit is R and $\frac{d\alpha}{dx}$ angular velocity of movable part, instead of the i current passing through the coil.

$i_i = -\frac{G}{R}.\frac{d\alpha}{dx}$ also an induction current passes

Since $d\emptyset = B.dS = B.S.d\alpha$

From Faraday's induction law $e = -N.\frac{d\emptyset}{dt}$, $e = -N.B.S\frac{d\alpha}{dt}$ can be written, when this value divided by R,

$i = \frac{e}{R} = -\frac{NBS}{R}.\frac{d\alpha}{dt}$ can be written

If mechanical damping is omitted, dynamic balance equation of the moving part.

$a.\frac{d^2\alpha}{dt^2} + c\alpha = G(i + i_i) = G(i - \frac{G}{R}.\frac{d\alpha}{dx})$ or

$$a.\frac{d^2\alpha}{dt^2} + \frac{G^2}{R}\frac{d\alpha}{dt} + c\alpha = Gi$$

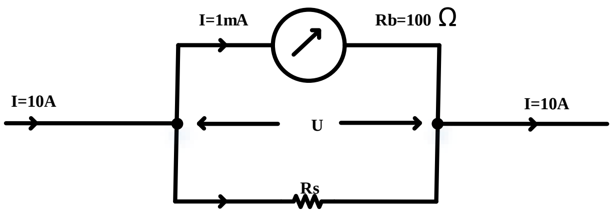
$\frac{G^2}{R}$ "damping coefficient" can be obtained electrically, $b_k = \frac{G^2}{R} = 2\sqrt{ac}$ is called critical damping coefficient if the rotating coil resistance is R_b, $R_{kd} = R_k - R_b$ resistance is called *critical external resistance* when the instrument is used, equivalent resistance of the outer circuit is made a bit less than this critical outer resistance.

Sensitivity of the moving coil instrument is $D = \frac{\Delta a}{\Delta i} = \frac{G}{c} = \frac{BSN}{c}$

As it is easy to do strong permanent magnets the sensitivity of the moving coil meters are so high and with 1pA current the pointer can make appearable deflection.

Specifications:

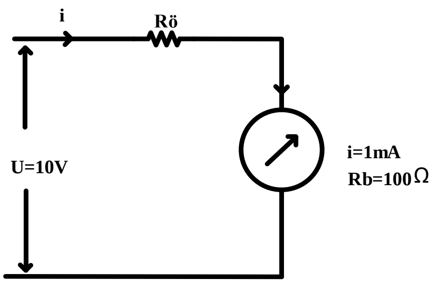
1. Their sensitivities are very high. Using permanent magnet greater induction can be obtained therefore the torque is greater. This makes the sensitivity greater.
2. As the current fed to the moving coil through the spirals greater currents cannot be measured directly. Measuring limits can be extended by using shunt resistance. Shunts are made of such materials that their resistance does not change with temperature (usually made of manganin)



I_1=0,999A

i.R_b=(I-i)R_s if the currents are rated.

$\frac{i}{I-i} = \frac{R_s}{R_b}$ Rs shunt resistance,



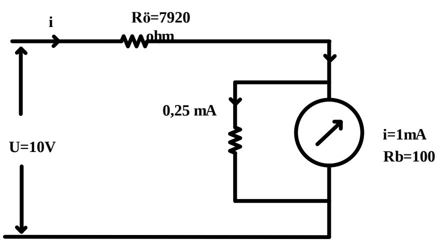
If a resistance serially connected to the moving coil a voltmeter can be done.

Moving coil meters can be used as voltmeters. Measuring limit can be more expanded by greater preresistance.

The value of $R_ö$, $R_ö = \frac{U}{i} - R_b$

If U=10V, $R_ö$=(10V)/(1mA)-100Ω= 9900Ω

By connecting a 9900 Ω serial preresistance connected to the moving coil a 10V voltmeter can be obtain but in practice a voltmeter can be done as follows



let us critical resistance be R_k= 500Ω

$b_k = \frac{G^2}{R_k} = 2\sqrt{ac},$ R_k=R_{kd}+R_b → R_{kd}=R_k-R_b= 500Ω-100Ω=400Ω

If the milliammeter passes 1mA current 400 Ω critical outer resistance passes 0,25mA. Therefore the current of the circuit is 1 mA+0,25mA=1,25mA

$$R_{eq} = \frac{400\Omega \ x \ 100\Omega}{(400+100)\Omega} = 80\Omega, \quad R_ö = \frac{9,9V}{1,25mA} = 7920\Omega$$

1,25mA x 80Ω=0,1V 10V-0,1V=9,9V obtained

3. Their sensitivity is very high. As moving part can be obtained very light in weight so that greater working torque can be achieved as a result, friction effect can be decreased greatly.

4. Their power dissipation is very low. Because greater working torque produced by a permanent magnet's magnetic field.
5. As moving coils are made of thin and sensitive wires and can loose their elasticity by temperature, these instruments can not be forced to charge with greater currents.

BALLISTIC GALVANOMETER

This is a moving coil meter that its inertia is increased as much as enough and is called ballistic galvanometer.

By this galvanometer very short time integrals of currents and voltages are measured i.e. electric flux which is rated with the electric charge shows a maximum deflection and magnetic flux can be measured.

$$(electric\ flux)\varphi = \int_0^\tau idt \ \ and\ (coil\ flux)\emptyset.N = \int_0^\tau edt$$

As it is seen the first equation is an electric charge amount and the second equation is flux-turn.

If the resistance of the circuit is R

$\varphi = \int_0^\tau idt = \int_0^\tau \frac{e}{R}dt$ can be written and therefore magnetic flux can be calculated.

The first condition for the accuracy of measurement the time τ, current passing through the galvanometer must be very small than the period of the galvanometer even before the moving coil starting to move discharging of current must be ended. Therefore, the inertia of the moving coil greater, control coefficient c smaller, the period of the movement can be increased.

$$T_o = 2\pi\sqrt{\frac{a}{c}} \gg \tau\ (period\ of\ the\ movement)$$

In order to grow the inertia momentum, while special galvanometers with large moving coil can be made, as well as, a weight can be handed to the moving part of some of the normally used galvanometers. These weights must be made of non-magnetizing materials.

For a very short time current passing through the galvanometer's coil affects it as if a pulse hits. This pulse gives it an initial velocity and by this velocity makes periodic or aperiodic movements and comes to zero.

Now, let calculate ω_0,

$$a\frac{d^2\alpha}{dt^2} + b\frac{d\alpha}{dt} + c\alpha = Gi$$

for a time τ, if a current flows through the coil, for $\frac{d\alpha}{dt} = \omega_0$ if the above equation is integrated during this time

$$a\int_0^\tau \frac{d^2\alpha}{dt^2}dt + b\int_0^\tau \frac{d\alpha}{dt}dt + c\int_0^\tau \alpha dt = G\underbrace{\int_0^\tau idt}_{Q}$$

$$a\frac{d\alpha}{dt} + b\alpha + c\alpha\tau = G.Q \qquad is\ obtained.$$

If during the τ time galvanometer's coil has not moved yet is accepted, in the equation $\alpha \cong 0$ can be taken. Therefore, initial velocity is

$$\omega_0 = \left.\frac{d\alpha}{dt}\right|_{t=\tau\to 0} = \frac{G}{a}.Q$$

(for $b\alpha=0$, $c\alpha\tau=0$) differantial equation of the movement of the moving coil from now on

$$a\frac{d^2\alpha}{dt} + b\frac{d\alpha}{dt} + c\alpha = 0 \ \ (as \ \ i = 0, Gi = 0)$$

for $t=\tau\cong 0$ $\alpha\cong 0$ and $\left.\frac{d\alpha}{dt}\right|_{t=\tau\to 0} = \omega_0 \approx \frac{G}{a}Q$

With these initial conditions if the equation is solved.

(for the case of $b^2<4ac$)

$a = \frac{2QG}{\sqrt{4ac-b^2}} e^{\frac{-bt}{2a}} . sin \frac{\sqrt{4ac-b^2}}{2a} t$ *can be obtained.*

If the first maximum deflection is calculated

$$\alpha_{1max} = \left[\underbrace{\frac{G}{\sqrt{ac}} e^{-\left(arctg\sqrt{\frac{4ac}{b^2}-1}\right) \Big/ \sqrt{\frac{4ac}{b^2}-1}}}_{\boxed{\text{Charge sensitivity}}} \right] . Q$$

If the charge sensitivity does not change, this ballistic deflection(temporary deflection) gives a criteria of the charge passing through the instrument.

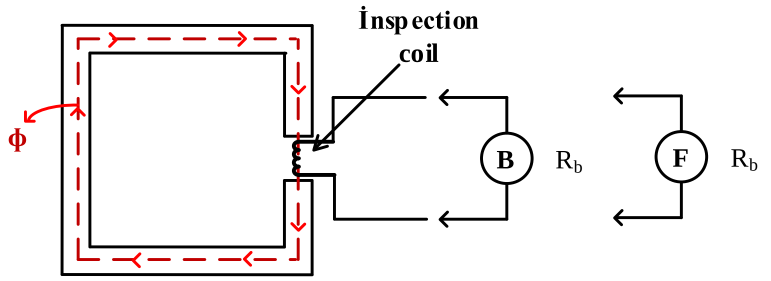


Figure. Measuring of magnetic flux by a ballistic galvonometer of fluxmeter

In the above figure, an inspection coil with the resistance R_m, that circulates ф flux N times is directly connected to the therminals of ballistic galvanometer.

R_b= coil resistance of galvanometer

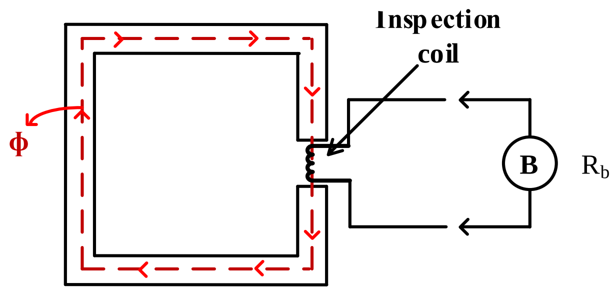
$R=R_b+R_m$ and for $b=G^2/R$ damping condition charge sensitivity let be D_q, either inspection coil is pull off as quick as from the magnetic circuit or the flux is made zero as quick as possible.

$$\alpha_{1max} = \frac{N}{R}.D_q.\emptyset \qquad\qquad \alpha_{1max} = D_\phi.\emptyset$$

D_ϕ = magnetic flux sensitivity of the ballistic galvanometer.

FLUXMETER

Fluxmeter is a moving coil instrument its main difference from the others is the current carrying spiral springs are very flexible and almost there is no control momentum (c≈0) coil resistance is small and its outer circuit resistance is also small. Therefore counter momentum of the movement is greater. In general outer circuit is an inspection coil.



In order to measure the ϕ flux either the coil is speedly withdrawn from the flux or the current of the electromagnet is decreased or cutoff. As there is no control momentum the pointer does not turn to zero, it stays where it was. Before a new measuring procedure in any way the pointer must be turned zero.

Movement equation (dynamic equation) of fluxmeter,

$$a\frac{d^2\alpha}{dt} + b\frac{d\alpha}{dt} = G.i \text{ by taking the integral of both sides}$$

(as c≈0   cy is omitted).

$$a\int_0^\tau \frac{d^2a}{dt^2}dt + b\int_0^\tau \frac{d\alpha}{dt}dt = G\underbrace{\int_0^\tau}_{Q} idt$$

The first term is the speed of system. The speed is zero before the movement starts and lasts. So,

$b.\alpha = G.Q$ this instrument measures the electric charge or electric flux.

Now, let us see, how this instrument measures the integrals of the current and voltage. To examine this, induced voltage across a coil with N turns is $e = N\frac{d\emptyset}{dt}$,   the current through the fluxmeter and the coil circuit is $N\frac{d\emptyset}{dt} = L\frac{di}{dt} + Ri$

where; L: self inductance of the whole circuit, R: the resistance of the whole circuit.

$i = \frac{N}{R}\frac{d\emptyset}{dt} - \frac{L}{R}\frac{di}{dt}$ when both sides are integrated

$$\int_0^\tau idt = \frac{N}{R}\int_0^\tau \frac{d\emptyset}{dt}dt - \frac{L}{R}\int_0^\tau \frac{di}{dt}dt$$

Suppose that at t=0, $\varphi = \emptyset, \quad \alpha = 0, \quad \omega = 0$

At t=τ $\varphi = 0, \quad \alpha = \alpha_0 \quad and \quad \omega = 0 \qquad again$

last term at t=0 and t=τ is zero

$$Q = \frac{N}{R}\int_{0}^{\tau} d\emptyset = \frac{N}{R}(\emptyset_\tau - \emptyset_0) \qquad \textit{is obtained}$$

It was $b\alpha = G.Q \rightarrow Q = \frac{b}{G}\alpha$ substituting $b\alpha = GQ$ into this equation $\frac{N}{R}(\emptyset_\tau - \emptyset_0 = \frac{b}{G}\alpha)$

If the mechanical damping is omitted $b \cong \frac{G^2}{R}$ can be written. Then

$$k = \frac{R}{N}\cdot\frac{b}{G} = \frac{R}{N}\cdot\frac{G^2}{RG} = \frac{G}{N} \qquad \text{obtained}$$

So, $\Delta\emptyset = \frac{G}{N}\alpha$ the result shows that fluxmeter measures both the voltage integral and flux changings. It is easy to read since the pointer stays on the final value. In ballistic galvanometers maximum value of the period is measured and when reading this value it is easy to make reading error.

THERMO INSTRUMENTS

In these instruments stable deflection position of the pointer depends on the temperature of a part of the instrument. The temperature of this part depends on average electric power of the instrument circuit and this power also depends on the current following from the circuit and voltage between the terminals.

There are two types of these instruments: **thermocouple** and **bimetallic** types

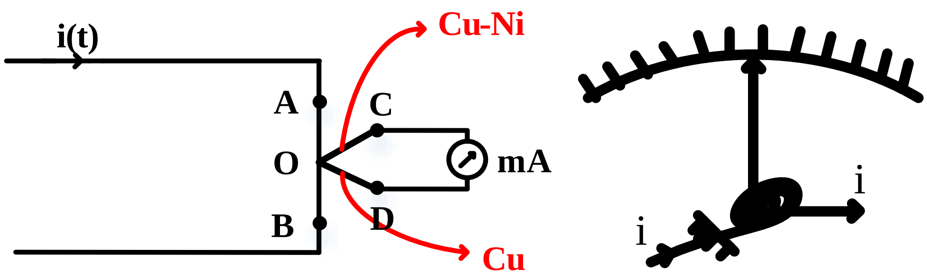


Figure (a) Structure of thermocouple and (b) bimetallic instrument.

In the figure (a) $\overline{OC}$ is a Cu-Ni alloy, $\overline{OD}$ is only copper. The temperature of these two parts is the same at point O, but are different at points C and D therefore the result of thermoelectric effect a current flows from the milliammeter. This deflection is given by $\alpha = kI^2$

where k: is a constant. This equation shows that this instrument can be used both in d.c and a.c.

In the figure(b) two different metals put upon each other in the shape of a spring. When a current passing through the spring the metals will be differently heated and will cause a bending or an expansion of the metals. A deflection of bimetallic instrument has the same equation as above. Only the k constants are different for the two kinds of the instruments.

MOVING FERROUS INSTRUMENTS

Operational principle of these instruments depend on the mechanical effect of the magnetic field on a ferrous core. As this effect is either “pushing” or “pulling” moving ferrous instruments are “pulling type ” and “pushing type” instruments.

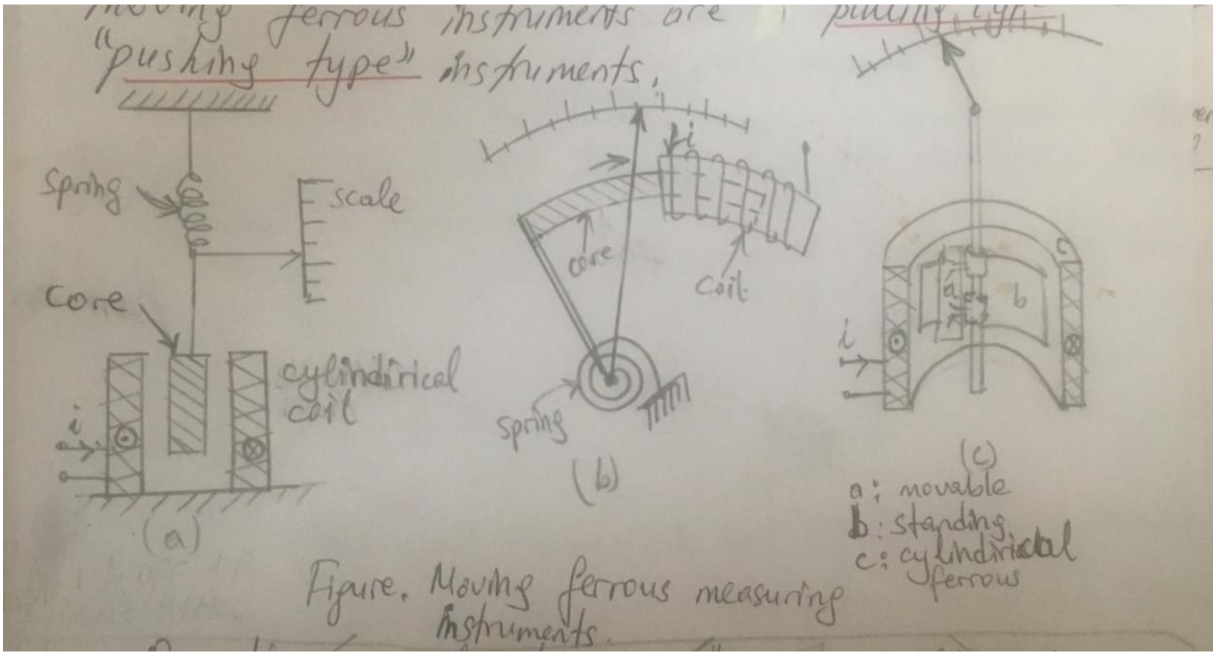


Figure. Moving ferrous measuring instruments.

When the ferrous part of the instrument moves within the coil, the self-inductance of the coil changes. Their self inductances are the function of the deflection amount α if the effective value of the current passing through the coil is I.

$$\alpha = \frac{1}{2c} . \frac{\partial L(\alpha)}{\partial \alpha} . I^2$$

and depends on self-inductance L, current I and c spring constant. If the ferrous core and the coil are made in proper configuration and $\frac{\partial L(\alpha)}{\partial \alpha} = \text{constant}$ is maintained the above equation becomes $\alpha = K.I^2$. the equation shows that this instrument can be used to measure both d.c and a.c. the quantity to be measured can either be proportional to the current or the self inductance then can be measured.

ELECTRODYNAMIC INSTRUMENTS

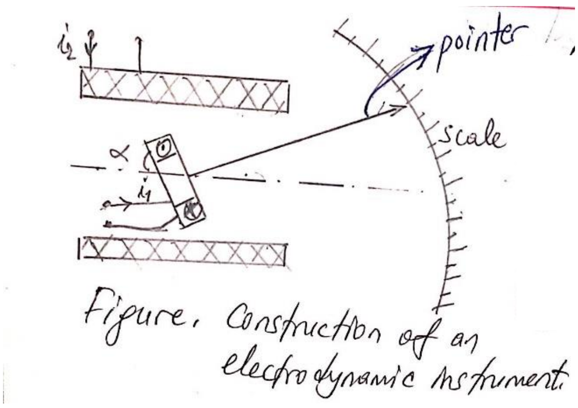


Figure. Construction of an electrodynamic instrument.

Operation principle depends on the magnetic fields produced by current carrying conductors exert a force on each other. These instruments consist of two coils one of which is standing and the other one is movable, are serially connected with each other. In the figure a simpler configuration is shown and coils are not serially connected. When the current $i_2(t)$ passes throught the outer coil, while $i_1(t)$ current passing through the inner coil, a torque affects to the pivot of the movable coil, its instant value is

$$m(t) = k i_1(t) . i_2(t) .$$

If an a.c. current to one coil and d.c. current to the other coil is applied average torque(momentum) is $\overline{M} = 0$ and instrument does not deflect, but if both coils are connected serially and d.c. is fed to these coils, the torque is

$$m(t) = kI^2, \qquad \overline{M} \neq 0$$

and the instrument works as a “**direct current ampermeter**”. If a voltage proportional to current I is applied to the coils, the last equation transforms and the instrument works as a “**direct current voltmeter**”.

When both coils are connected serially an alternating current I (effective value) passes through the coils

$$m(t) = kI^2, \qquad \overline{M} \neq 0$$

equation is valid and the instrument work as an “**alternating current ampermeter**”. Similarly if a voltage has an effective value U is applied, in this case, the instrument works as an “**alternating current voltmeter**.” As I_1 and I_2 are effective values and $\overline{M}$ average momentum permanent deflection is

$$\alpha = \frac{\overline{M}}{c} = \frac{K}{c}.I_1.I_2.cos\varphi$$

Where, ϑ is the phase difference between I_1 and I_2. If a voltage which is proportional to current is applied to the movable coil $I_1 = \frac{U}{R}$ the deflection is

$$\alpha = \frac{K}{cR}.U.I_2.cos\varphi$$

On the other hand, it is well known that active power is

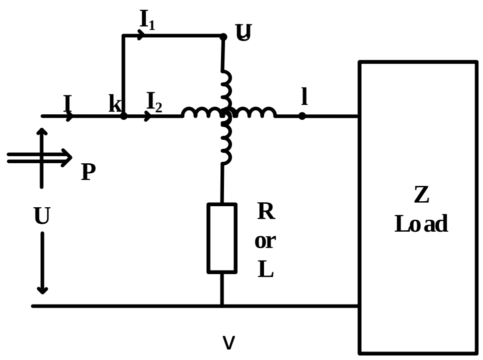
$$P = U.I_2.cos\varphi$$

so the deflection is

$$\alpha = \frac{K}{cR}.P$$

proportional to the active power. This is to show that this instrument can measure the power. In practice, R is greater, therefore I_1 is very small so, $I = I_1 + I_2 \approx I_2$ can be taken, from this result is

$$P = U.I.cos\varphi$$



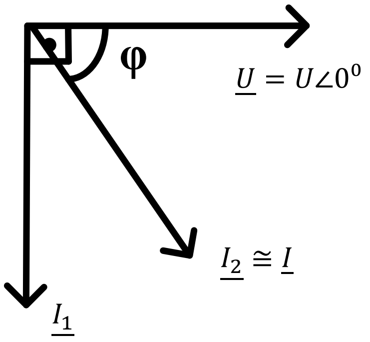
if this instrument is connected as in the figure below, it works as a wattmeter.

k-l: current terminals

u-v: voltage terminals

When instead of R resistance a coil with a great self inductance (greater N number of turns and made of a thin wire) is used. A phasor diagram as in the figure below.

Figure : coil connection to use an electrodynamic measuring instrument as a wattmeter.



As it is seen from the figure

$\underline{I_1} = \frac{Y}{\omega L} e^{-j\frac{\pi}{2}}$, $I_1 = \frac{U}{\omega L}$, $I_2 = \imath$ and $I_1\angle I_2 = 90^0 - \varphi$

Reminding $[\cos(90^0 - \varphi) = cos90.cos\varphi + sin90.sin\varphi]$

$\cos(90^0 - \varphi) = sin\varphi$

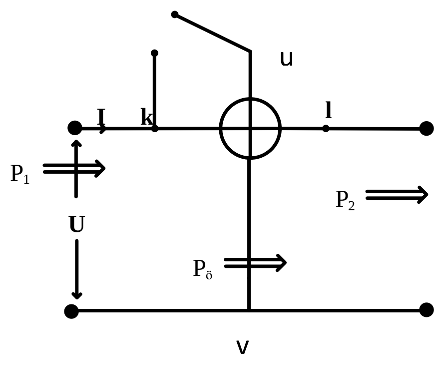
deflection amount $\alpha = \frac{k}{c\omega L} UIsin\varphi = \frac{k}{c\omega L} Q$

is obtained. It means that the deflection is proportional to the reactive power. So this instrument operates as a VARmeter.

R and L are built-in during the manufacturing of the instrument. From the equation it is seen that the deflection of wattmeter is not a function of ω, on the other hand the deflection of VARmeter is a function of ω therefore consequently is a function of line frequency.

In the electrodynamic instruments which are designed to measure power, standing coil is affected by current, movable coil is affected by voltage and these instruments are called wattmeters. The instrument has two pairs of terminals, k-l are current terminals, u-v are voltage terminals.

In a two-wire transmission the to measure transmitted power from left to right a wattmeter can be connected to the circuit in two ways as follow,



$$P_{ö1} = P_2 + P_a = P_1 - P_g$$

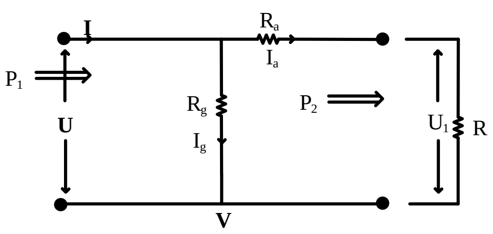
$$P_{ö2} = P_2 + P_g = P_1 - P_a$$

$P_ö$: measured power

P_1 : power to be transmitted from left to right at the beginning of the line

P_2: transmitted power or power received by load.

u terminal might be connected to k or l terminals. Only the standing coil of the instrument is connected between k and l terminals and to the k terminal a point ⊙ or a star sign * is put and the other one is left without any sign.



Case:1) **Voltage coil is connected before the current coil**

in this case measured power is $P_ö = I_a^2(R_a + R)$ because

Equivalent circuit

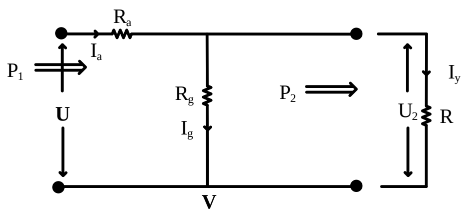
$$P_ö = U.I_a = \underbrace{I_a(R_a + R)}_{U}.I_a = I_a^2(R_a + R)$$

$$P_{ö1} = I_a^2(R_a + R) \rightarrow \underbrace{I_a^2 R_a}_{P_a} + \underbrace{I_a^2 R}_{P_2}$$

Thus $P_{ö1} = P_2 + P_a$

At the same time, $P_{ö1} = U(I - I_g) = UI - UI_g = UI - UI_g = P_1 - P_g$

Case:2) **Current coil is connected before the voltage coil.**



$$P_ö = I_a.U_2,\; I_a = I_g + I_y,\; P_{ö2} = (I_g + I_y)U_2$$

$$P_{ö2} = \underbrace{\frac{U_2^2}{Rg}}_{P_g} + \underbrace{\frac{U_2^2}{Ry}}_{P_2}$$

At the same time

$$P_{ö2} = P_1 - P_a$$

Rule:1) Always $P_2 < P_ö < P_1$

P_2: power received by load, $P_ö$: measured power, P_1: incident power

Power measured by a wattmeter

$$P = U_N.I_N.\frac{d}{D} \; [W]$$

Where, U_N: nominal value of the voltage coil,

I_N: nominal value of the current coil,

d : deflection of the pointer,

D= full scale deflection of the pointer.

At the same time, P=U.I.cosφ [W]

Where, U: effective value of the line voltage

I : effective value of the current

φ : phase difference between voltage and current.

Rule: 2) Wattmeters show the product of the voltage of voltage circuit and current of the current circuit

Rule: 3) In case of very small cosφ, because of highly inductive and capacitive circuits, when wattmeters are loaded more than nominal current and nominal voltage values, the pointer may deflects a small value but one or both of the coils may be broken down(fired).

Therefore wattmeter circuit should not be loaded greater than the 1,5 times nominal currents and nominal voltages, for a long time.

Rule: 4) If the current circuit of the wattmeter is connected to the u terminal of the voltage circuit, wattmeter measures the power passes from terminal k to l. But if the power flowing in the reverse direction the pointer deflects in reverse direction and strikes the counter movement pin