

1) $y = \sin x^{\tan x}$, $\ln y = \tan x \ln \sin x$

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan x \ln \sin x = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{\cos x} \ln \sin x = \lim_{x \rightarrow \frac{\pi}{2}} \sin x \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln \sin x}{\cos x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{-\sin x} = - \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\sin^2 x} = 0 \quad \checkmark 5p$$

$$\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x} = e^{\lim_{x \rightarrow \frac{\pi}{2}} \tan x \ln \sin x} = e^0 = 1 \quad \checkmark 5p$$

2) $-f'(x) = \frac{1}{(2-x)^2}$, $-f''(x) = \frac{2}{(2-x)^3}$, $-f'''(x) = \frac{2 \cdot 3}{(2-x)^4}$, ... $\checkmark 5p$

$$f^{(n)}(x) = \frac{n!}{(2-x)^{n+1}} \quad \checkmark 5p$$

3) i) $TK = \mathbb{R} - \{1, -1\}$

ii) $\lim_{x \rightarrow 1^-} \frac{x}{1-x^2} = \infty$
 $x=1$ doğrusu düzey asim.

$\lim_{x \rightarrow -1^-} \frac{x}{1-x^2} = +\infty$
 $x=-1$ doğrusu düzey asim.

$\lim_{x \rightarrow 1^+} \frac{x}{1-x^2} = -\infty$

$\lim_{x \rightarrow -1^+} \frac{x}{1-x^2} = -\infty$

$\lim_{x \rightarrow \pm \infty} \frac{x}{1-x^2} = 0$, $y=0$ doğrusu yatay asim.

iii) $x=0$ için $y=0$ $\checkmark 5p$

iv) $f'(x) = \frac{1-x^2 - x(-2x)}{(1-x^2)^2} = \frac{1-x^2+2x^2}{(1-x^2)^2} = \frac{x^2+1}{(1-x^2)^2}$

$x=-1$ ve $x=1$ de $-f'$ mevcut deg.?

(2)

v)

x	$-\infty$	-1	0	1	$+\infty$
f'	+	+	+	+	
f	0 $\nearrow +\infty$	$-\infty \nearrow$	0 $\nearrow +\infty$	$-\infty \nearrow$	0

✓ 5p

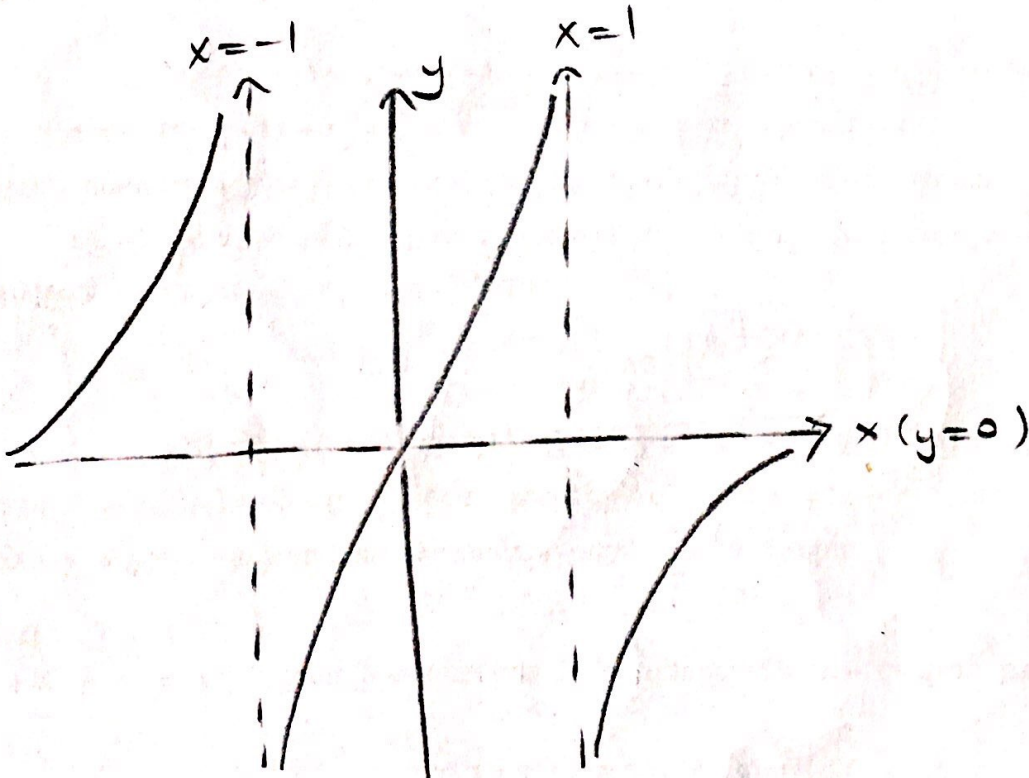
vi)
$$f''(x) = \frac{2x(1-x^2)^2 - (x^2+1)2 \cdot (-2x)}{(1-x^2)^4} = \frac{2x(x^4+3)}{(1-x^2)^4}$$

$$f''(0) = 0$$

x	-1	0	1	
f''	-	-	+	+
f				

$x=0$ da f fonksiyonu
bir dönüm noktasına
sahiptir.

vii)



✓ 5p

4) A) $\ln x = t \quad \frac{1}{x} dx = dt \quad \checkmark 5p$

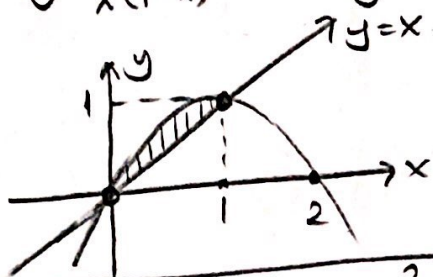
$\int \cos t dt = \sin t + C = \sin(\ln x) + C \quad \checkmark 5p$

B) $\frac{x-2}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$

$A - Ax + Bx = x - 2$
 $-A + B = 1 \quad \left\{ \begin{array}{l} B = -1 \\ A = -2 \end{array} \right. \quad \checkmark 5p$

$\int \frac{x-2}{x(1-x)} dx = \int \frac{-2dx}{x} + \int \frac{-dx}{1-x} = -2 \ln|x| + \ln|1-x| + C \quad \checkmark 5p$

5)



Ortak çizim: $x = 2x - x^2$
 $x^2 - x = 0 \Rightarrow x(x-1) = 0$
 $x = 0 \vee x = 1$

A) $A_B = \int_0^1 (2x - x^2 - x) dx \quad \checkmark 5p$

$= \int_0^1 (x - x^2) dx = \left. \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1$

$= \frac{1}{6} \quad \checkmark 5p$

B) $\pi \int_0^1 [(2x - x^2)^2 - x^2] dx = \pi \int_0^1 (4x^2 - 4x^3 + x^4 - x^2) dx \quad \checkmark 10$

$= \pi \left(\frac{x^5}{5} - x^4 + x^3 \right) \Big|_0^1 = \frac{\pi}{5} \quad \checkmark 5p$

6) $c_n = \frac{(-1)^n}{n^2}$, $\lim \sqrt[n]{|c_n|} = \lim \frac{1}{\sqrt[n]{n^2}} = 1 \quad \checkmark 5p$

$|x-2| < 1 \Rightarrow 1 < x < 3 \quad \checkmark 5p$

$x=1$ için $\sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow$ yakıwak

$x=3$ için $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \rightarrow$ yakıwak

Yakıwaklık aralığı $[1, 3] \quad \checkmark 5p$