CEVAP ANAHTARI (17.06.2019)

1)
$$\begin{array}{c} -3 \\ -2 \end{array}\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & -1 & 2 & 1 \\ 3 & 1 & 3 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -5 & 0 & -5 \\ 0 & -5 & 0 & -5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -5 & 0 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdot -1/5$$

✓5p

$$-2 \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x+z=1 \\ y=1 \end{array}$$ ✓5p

$x=1-z,\ y=1,\ z=z,\ z\in\mathbb{R}$. ✓5p

2) $\nabla f=\left(2xy+\frac{1}{x}\right)\vec{i}+(x^2-z)\vec{j}+\left(\frac{1}{z}-y\right)\vec{k}$ ✓5p

$\nabla f\big|_{P_0}=(-4+1)\vec{i}+(1-1)\vec{j}+(1+2)\vec{k}=-3\vec{i}+3\vec{k}$ ✓5p

$(D_{\vec{u}}f)(P_0)=(-3\vec{i}+3\vec{k})\cdot\vec{u}=-1+2=1$ ✓5p

3) $g(x,y)=2x+y-5=0$

$\nabla f=\lambda\nabla g\ ,\quad g(x,y)=0$

$2x\vec{i}+2y\vec{j}=\lambda(2\vec{i}+\vec{j})$ ✓5p $\Rightarrow\ x=\lambda,\ y=\frac{\lambda}{2}$ ve $2x+y=5$ ✓5p

$2\lambda+\frac{\lambda}{2}=5\ \Rightarrow\ \frac{5\lambda}{2}=5\ \Rightarrow\ \lambda=2$ ✓5p

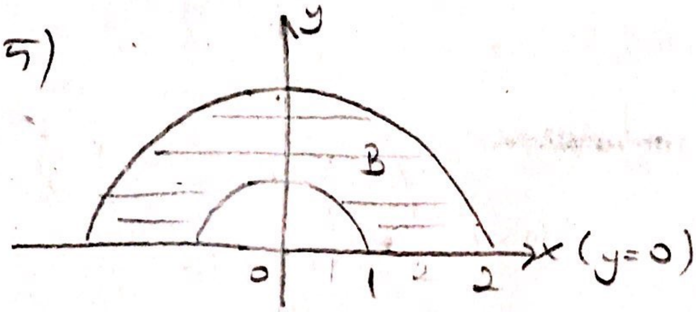
Ekstremum nokta (2,1) koordinatlı noktadır. ✓5p

4) $\displaystyle\int_0^2\int_0^{4-y^2} y\,dx\,dy=\int_0^2 xy\Big|_0^{4-y^2}dy=\int_0^2(4-y^2)y\,dy$ ✓10p

$\displaystyle=\int_0^2(4y-y^3)\,dy=2y^2-\frac{y^4}{4}\Big|_0^2$

$=8-4=4$ ✓5p

5)



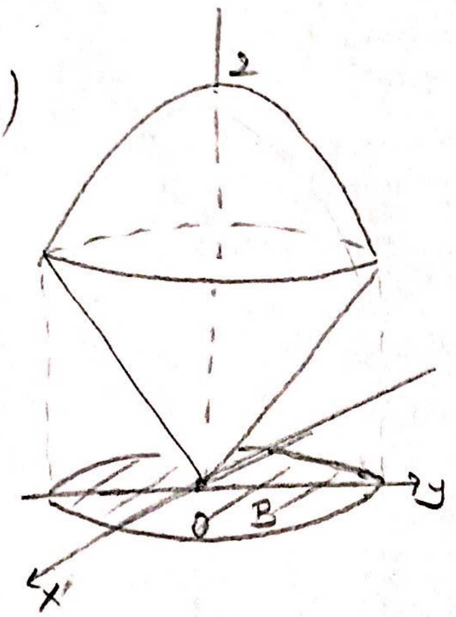
$$A = \iint_B dx\,dy$$

$$= \int_0^{\pi}\int_1^{2} r\,dr\,d\theta \quad \checkmark\ 10p$$

$$= \int_0^{\pi} \left(\frac{r^2}{2} \Big|_1^2 \right) d\theta$$

$$= \int_0^{\pi} \frac{3}{2}\,d\theta = \frac{3}{2}\pi \quad \checkmark\ 5p$$

6)

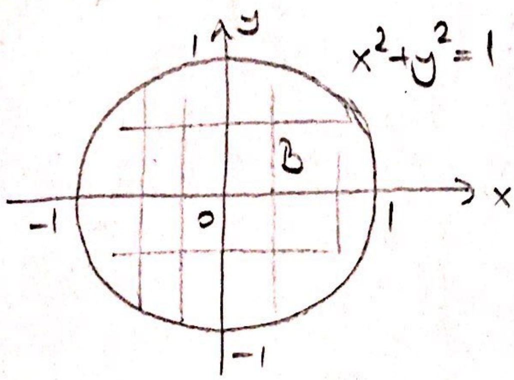


$$z = 2 - z^2$$

$$\Rightarrow z^2 + z - 2 = 0$$

$$(z+2)(z-1) = 0$$

$$\cancel{z=-2} \ \vee \ z = 1 \quad \checkmark\ 5p$$



$$V = \iint_B \left[\int_{\sqrt{x^2+y^2}}^{2-x^2-y^2} dz \right] dx\,dy \quad \checkmark\ 5p$$

$$= \iint_B 2 - x^2 - y^2 - \sqrt{x^2+y^2}\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^1 (2 - r^2 - r)\, r\,dr\,d\theta \quad \checkmark\ 5p$$

$$= \int_0^{2\pi} r^2 - \frac{r^4}{4} - \frac{r^3}{3} \Big|_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{5}{12}\,d\theta = \frac{5}{6}\pi \quad \checkmark\ 5p$$