① $(x^2-y^2e^{y/x})dx+(x^2+xy)e^{y/x}dy=0$

$$\frac{dy}{dx}=\underbrace{\frac{y^2e^{y/x}-x^2}{(x^2+xy)e^{y/x}}}_{\to f(x,y)}\Rightarrow f(tx,ty)=\frac{t^2y^2e^{ty/tx}-t^2x^2}{(t^2x^2+txty)e^{ty/tx}}=f(x,y)$$

↳ Homojen Denklem

$$v=\frac{y}{x}\Rightarrow y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{v^2x^2e^{vx/x}-x^2}{(x^2+vx^2)e^{vx/x}}=\frac{v^2e^v-1}{(1+v)e^v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{v^2e^v-1}{e^v+ve^v}-v=\frac{\cancel{v^2e^v}-1-ve^v-\cancel{v^2e^v}}{e^v(1+v)}$$

$$\Rightarrow\int\frac{e^v(1+v)}{-1-ve^v}dv=\int\frac{dx}{x}\Rightarrow-\int\frac{du}{u}=\ln|x|+c$$

$$\Rightarrow-\ln|1+ve^v|=\ln|x|+c$$

$$\xRightarrow{v=y/x}-\ln\left|1+\frac{y}{x}e^{y/x}\right|=\ln|x|+c$$ //

$1+ve^v=u$

$(e^v+ve^v)dv=du$

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② $y'+3\frac{y}{x}=-3\frac{e^x}{x}y^{2/3}$ → Bernoulli denklemi

$$v=y^{1-\frac{2}{3}}=y^{1/3}\Rightarrow\frac{dv}{dx}=\frac{1}{3}y^{-2/3}\cdot\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=3y^{2/3}\frac{dv}{dx}$$

$$3y^{2/3}\frac{dv}{dx}+3\frac{y}{x}=-3\frac{e^x}{x}y^{2/3}$$

$$\Rightarrow\frac{dv}{dx}+\frac{1}{x}v=-\frac{e^x}{x}\Rightarrow u(x)=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$$

$$\Rightarrow\frac{d}{dx}(x.v)=-e^x\Rightarrow x.v=-\int e^xdx=-e^x+c$$

$$\Rightarrow x.y^{1/3}=-e^x+c$$ //

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③ $y''-\frac{y'}{x-1}=0\Rightarrow v=y'\Rightarrow\frac{dv}{dx}=y''$

$$\frac{dv}{dx}=\frac{v}{x-1}\Rightarrow\int\frac{dv}{v}=\int\frac{dx}{x-1}\Rightarrow\ln|v|=\ln|x-1|+c_1$$

$$\Rightarrow|v|=|x-1|.e^{c_1}$$

$$\Rightarrow v=c(x-1),\quad c:=\pm e^{c_1}$$

$$\Rightarrow\frac{dy}{dx}=c(x-1)$$

$$\Rightarrow\int dy=\int c(x-1)dx\Rightarrow y=c\left(\frac{x^2}{2}-x\right)+c$$

④ $(2xy^2 - y\sin x + 2x + 1)\,dx + (2x^2y + \cos x + y^{-1})\,dy = 0$

$M_y = 4xy - \sin x \;=\; N_x = 4xy - \sin x \rightarrow$ Tam Dif. Denk.

$F_x = M \Rightarrow F(x,y) = \int M\,dx = \int (2xy^2 - y\sin x + 2x + 1)\,dx$

$= x^2y^2 + y\cos x + x^2 + x + h(y) = c$

$\xRightarrow{F_y = N} \cancel{2x^2y} + \cancel{\cos x} + h'(y) = \cancel{2x^2y} + \cancel{\cos x} + \frac{1}{y}$

$\Rightarrow h'(y) = \frac{1}{y} \Rightarrow h(y) = \int \frac{dy}{y} = \ln|y| + c_1$

$\Rightarrow F(x,y) = x^2y^2 + y\cos x + x^2 + x + \ln|y| = C \quad , \quad C := c - c_1$

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⑤ $y'' - 6y' + 9y = x + \sin 2x \rightarrow$ Belirsiz Katsayılar yönt.

$\alpha^2 - 6\alpha + 9 = 0 \Rightarrow (\alpha - 3)^2 = 0 \Rightarrow \alpha_1 = \alpha_2 = 3$

$u = (c_1 + c_2 x)\,e^{3x}$

$f_1(x) = x \rightarrow V_1 = A_0 + A_1 x$

$f_2(x) = \sin 2x \rightarrow V_2 = B_0 \cos 2x + C_0 \sin 2x$

$\Rightarrow f(x) = x + \sin 2x \rightarrow V = V_1 + V_2 = A_0 + A_1 x + B_0 \cos 2x + C_0 \sin 2x$

$V' = A_1 - 2B_0 \sin 2x + 2C_0 \cos 2x$

$V'' = -4B_0 \cos 2x - 4C_0 \sin 2x$

$\Rightarrow -4B_0 \cos 2x - 4C_0 \sin 2x - 6A_1 + 12B_0 \sin 2x - 12C_0 \cos 2x$
$+ 9A_0 + 9A_1 x + 9B_0 \cos 2x + 9C_0 \sin 2x = x + \sin 2x$

$\Rightarrow -4B_0 - 12C_0 + 9B_0 = 0 \Rightarrow 5B_0 - 12C_0 = 0 \quad (\cdot 12)$
$-4C_0 + 12B_0 + 9C_0 = 1 \Rightarrow 5C_0 + 12B_0 = 1 \quad (\cdot -5)$

$-144C_0 - 25C_0 = -5$

$C_0 = \frac{5}{169}, \quad B_0 = \frac{12}{169}$

$9A_1 = 1 \Rightarrow A_1 = \frac{1}{9}$

$-6A_1 + 9A_0 = 0 \Rightarrow A_0 = \frac{\cancel{6}\,2}{9 \cdot \cancel{9}\,3} = \frac{2}{27}$

$\Rightarrow V = \frac{2}{27} + \frac{1}{9}x + \frac{12}{169}\cos 2x + \frac{5}{169}\sin 2x$

$\Rightarrow y = u + V = (c_1 + c_2 x)e^{3x} + \frac{2}{27} + \frac{x}{9} + \frac{12}{169}\cos 2x + \frac{5}{169}\sin 2x$

⑥ $y'' - 2y' + y = \frac{e^x}{1+x^2}$ → Değişen Parametreler Yöntemi

$\alpha^2 - 2\alpha + 1 = 0 \Rightarrow (\alpha - 1)^2 = 0 \Rightarrow \alpha_1 = \alpha_2 = 1$

$u = (c_1 + c_2 x)\, e^x \Rightarrow y_1 = e^x,\ y_2 = x e^x$

$$-/\ v_1' e^x + v_2' x e^x = 0$$

$$v_1' e^x + v_2' e^x + v_2' x e^x = \frac{e^x}{1+x^2}$$

$$+ \quad \overline{\qquad\qquad\qquad\qquad\qquad}$$

$$v_2' \cancel{e^x} = \frac{\cancel{e^x}}{1+x^2} \Rightarrow v_2 = \int \frac{dx}{1+x^2} = \arctan x$$

$$v_1' e^x + \frac{x}{1+x^2} e^x = 0 \Rightarrow v_1' = -\frac{x}{1+x^2} \Rightarrow v_1 = -\int \frac{x}{1+x^2} dx$$

$$= -\int \frac{du}{2u} = -\frac{1}{2} \ln(1+x^2)$$

$\Rightarrow V = v_1 y_1 + v_2 y_2 = -\frac{1}{2} \ln(1+x^2) e^x + x(\arctan x) e^x$

$\Rightarrow y = u + V = (c_1 + c_2 x) e^x - \frac{1}{2} \ln(1+x^2) e^x + x(\arctan x) e^x$