Ad-Soyad:

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Bölüm: Elektrik-Elektronik Mühendisliği

* Her soru 20 puan! Süre 80 dk!

DİFERANSİYEL DENKLEMLER (MAT2011) ARASINAV SORULARI

1) $xy' + y = y^2x^2lnx$, $x > 0$ diferansiyel denkleminin genel çözümünü bulunuz.

$y' + \frac{1}{x}y = y^2 x \ln x \Rightarrow v = y^{1-2} = y^{-1} \Rightarrow \frac{dv}{dx} = -y^{-2}\frac{dy}{dx}$ (5)

$\Rightarrow \frac{dy}{dx} = -y^2\frac{dv}{dx} \Rightarrow -y^2\frac{dv}{dx} + \frac{1}{x}y = y^2 x \ln x$ (5)

$\Rightarrow \frac{dv}{dx} - \frac{1}{x}v = -x\ln x \Rightarrow u(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$\Rightarrow \frac{d}{dx}\left(\frac{1}{x}\cdot v\right) = -\ln x$ (5) $\Rightarrow \frac{1}{x}\cdot v = -\int \ln x\, dx$

$\ln x = u \quad dx = dv$

$\frac{1}{x}dx = du \quad x = v$

$\Rightarrow \frac{1}{x}\cdot v = -x\ln x + \int dx = -x\ln x + x + c$

$\Rightarrow v = -x^2\ln x + x^2 + cx$

$\Rightarrow y^{-1} = -x^2\ln x + x^2 + cx$ (5)

2) $\underbrace{(2xy - \frac{y^3}{x^2})}_{M}dx + \underbrace{\left(x^2 + \frac{3y^2}{x}\right)}_{N}dy = 0$ diferansiyel denkleminin genel çözümünü bulunuz.

$My = 2x - \frac{3y^2}{x^2}$, $Nx = 2x - \frac{3y^2}{x^2} \Rightarrow My = Nx$ (5)

$F_x = M \Rightarrow F = \int M dx = \int (2xy - \frac{y^3}{x^2})dx = x^2y + \frac{y^3}{x} + h(y)$ (5)

$F_y = N \Rightarrow x^2 + \frac{3y^2}{x} + h'(y) = x^2 + \frac{3y^2}{x}$ (5) $\Rightarrow h'(y) = 0$

$\Rightarrow h(y) = c_1 \Rightarrow F = x^2y + \frac{y^3}{x} = C.$

(5)

3) $e^{-x}(1 + \frac{dy}{dx}) = 1, y(0) = 2$ başlangıç değer probleminin çözümünü bulunuz.

$1 + \frac{dy}{dx} = e^x \Rightarrow \frac{dy}{dx} = e^x - 1 \Rightarrow dy = (e^x - 1)dx$ (5)

$\Rightarrow y = e^x - x + c$ (5)

$y(0) = 2 \Rightarrow 2 = 1 - 0 + c \Rightarrow c = 1$ (5)

$\Rightarrow y = e^x - x + 1$ (5)

4) $(xsin\frac{y}{x} - ycos\frac{y}{x})dx + (xcos\frac{y}{x})dy = 0$ diferansiyel denkleminin genel çözümünü bulunuz.

$$x\cos\frac{y}{x}\,dy = \left(-x\sin\frac{y}{x} + y\cos\frac{y}{x}\right)dx$$

$$\Rightarrow \frac{dy}{dx} = -\tan\frac{y}{x} + \frac{y}{x} \quad \Rightarrow f(tx,ty) = -\tan\frac{ty}{tx} + \frac{ty}{tx}$$

$$= -\tan\frac{y}{x} + \frac{y}{x} = f(x,y)$$

$f(x,y)$ (5)

$\Rightarrow$ Homojen! $v = \frac{y}{x} \Rightarrow y = vx \Rightarrow y' = v + v'x$

$$\Rightarrow v + v'x = -\tan v + v \quad \Rightarrow v'x = -\tan v \quad (5)$$

$$\Rightarrow \frac{dv}{\tan v} = -\frac{dx}{x} \Rightarrow \int \cot v\,dv = -\int \frac{dx}{x}$$

$\sin v = u$
$\Rightarrow \cos v\,dv = du$

$$\Rightarrow \int \frac{du}{u} = -\int \frac{dx}{x} \Rightarrow \ln|u| = -\ln|x| + c$$

$$\Rightarrow \ln|\sin v| = -\ln|x| + c \quad (5)$$

$$\Rightarrow \ln|\sin v| + \ln|x| = c \Rightarrow \ln\left|x.\sin\frac{y}{x}\right| = c \quad (5)$$

$$\Rightarrow x\sin\frac{y}{x} = C\,, \quad C := \pm e^{c}.$$

5) $y'' - 2y' + y = e^x + 4$ diferansiyel denkleminin genel çözümünü bulunuz.

$y''-2y'+y=0 \Rightarrow \alpha^2-2\alpha+1=0 \Rightarrow \alpha_1=\alpha_2=1$

$\Rightarrow u = (c_1 + c_2x)e^x$ (5)

$f_1(x) = e^x \Rightarrow P(x)=1, \alpha=1 \Rightarrow k=2, n=0$

$V_1 = x^2.A_0e^x$

$f_2(x) = 4 \Rightarrow P(x)=4, \alpha=0 \Rightarrow k=0, n=0$

$V_2 = B_0$

$\Rightarrow V = V_1+V_2 = A_0x^2e^x + B_0$ (5)

$V' = 2A_0xe^x + A_0x^2e^x$

$V'' = 2A_0e^x + 2A_0xe^x + 2A_0xe^x + A_0x^2e^x$

$\Rightarrow 2A_0e^x + 4A_0xe^x + A_0x^2e^x - 4A_0xe^x - 2A_0x^2e^x$

$+A_0x^2e^x + B_0 = e^x + 4$

$\Rightarrow 2A_0 = 1 \Rightarrow A_0 = \frac{1}{2}\,, \quad B_0 = 4$

$V = \frac{1}{2}x^2e^x + 4$ (5)

$\Rightarrow y = (c_1+c_2x)e^x + \frac{1}{2}x^2e^x + 4$ (5)