

$$1) \begin{bmatrix} -1 & 2 & 1 & 5 & -1 \\ 1 & 1 & 2 & 7 & 4 \\ 3 & 0 & 1 & 7 & 5 \end{bmatrix} \sim \begin{bmatrix} 0 & 3 & 3 & 12 & 3 \\ 1 & 1 & 2 & 7 & 4 \\ 0 & -3 & -5 & -14 & -7 \end{bmatrix}$$

$$\frac{1}{3} \sim \begin{bmatrix} 1 & 1 & 2 & 7 & 4 \\ 0 & 3 & 3 & 12 & 3 \\ 0 & -3 & -5 & -14 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 7 & 4 \\ 0 & 1 & 1 & 4 & 1 \\ 0 & -3 & -5 & -14 & -7 \end{bmatrix} \begin{matrix} \nearrow \\ \nwarrow \end{matrix}$$

$$-\frac{1}{2} \sim \begin{bmatrix} 1 & 0 & 1 & 3 & 3 \\ 0 & 1 & 1 & 4 & 1 \\ 0 & 0 & -2 & -2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 3 & 3 \\ 0 & 1 & 1 & 4 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix} \begin{matrix} \nearrow \\ \nwarrow \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} x + 2t &= 1 \\ y + 3t &= -1 \\ z + t &= 2 \end{aligned} \Rightarrow \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1-2t \\ -1-3t \\ 2-t \\ t \end{bmatrix}, t \in \mathbb{R}.$$

$$2) \begin{vmatrix} \lambda & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 \\ -1 & 0 & 3 & 1 \\ 3 & 1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} \lambda & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 \\ -1 & -3 & 2 & 0 \\ 3 & 1 & 2 & 0 \end{vmatrix} = 1 \cdot (-1)^{2+4} \begin{vmatrix} \lambda & 0 & 1 \\ -1 & -3 & 2 \\ 3 & 1 & 2 \end{vmatrix}$$

$$\stackrel{2}{=} \begin{vmatrix} \lambda & 0 & 1 \\ -1 & -3 & 2 \\ 3 & 1 & 2 \end{vmatrix} = \begin{vmatrix} \lambda & 0 & 1 \\ 8 & 0 & 8 \\ 3 & 1 & 2 \end{vmatrix} = 1 \cdot (-1)^{3+2} \begin{vmatrix} \lambda & 1 \\ 8 & 8 \end{vmatrix}$$

$$= (-1)(8\lambda - 8) = 8 - 8\lambda$$

$$\det A = 0 \quad \text{ için } \quad 8 - 8\lambda = 0 \Rightarrow \lambda = 1 //$$

$$3) \vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 3 \\ 4 & 1 & 2 \end{vmatrix} = -3\vec{i} + 10\vec{j} + \vec{k} \quad \text{olduğundan düzlemin denk}$$

$$-3x + 10y + z + D = 0 \quad \text{ dir. } \quad (-3) \cdot 1 + 10 \cdot 2 + (-3) + D = 0 \Rightarrow D = -14$$

$$-3x + 10y + z - 14 = 0$$

$$4) \alpha'(t) = \frac{d\alpha}{dt} = -\frac{4}{5} \sin t \vec{i} - \cos t \vec{j} + \frac{3}{5} \sin t \vec{k}$$

$$\alpha''(t) = \frac{d^2\alpha}{dt^2} = -\frac{4}{5} \cos t \vec{i} + \sin t \vec{j} + \frac{3}{5} \cos t \vec{k}$$

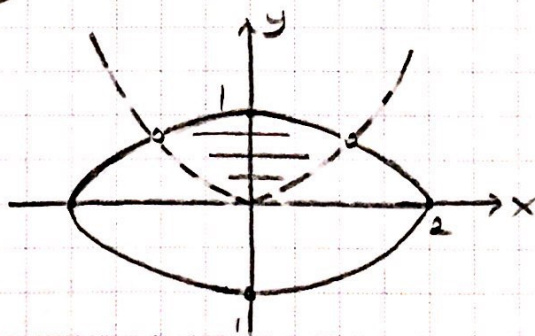
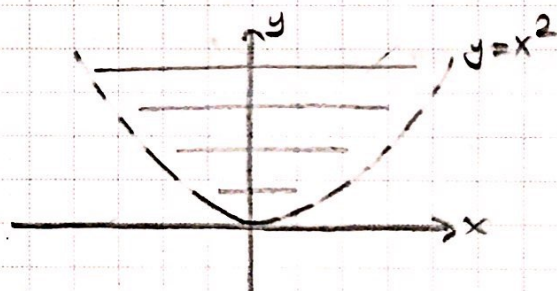
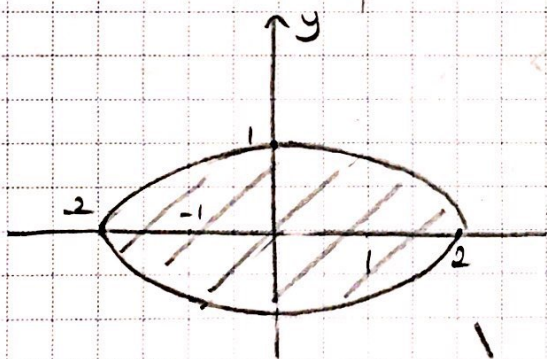
$$\|\alpha'(t)\| = \sqrt{\frac{16}{25} \cos^2 t + \sin^2 t + \frac{9}{25} \cos^2 t} = 1$$

$$\alpha'(t) \times \alpha''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{4}{5} \sin t & -\cos t & \frac{3}{5} \sin t \\ -\frac{4}{5} \cos t & \sin t & \frac{3}{5} \cos t \end{vmatrix}$$

$$= -\frac{3}{5} \vec{i} - \frac{4}{5} \vec{k}$$

$$\|\alpha'(t) \times \alpha''(t)\| = \sqrt{\frac{9}{25} + \frac{16}{25}} = 1 \Rightarrow K(t) = \frac{1}{1/3} = 1 //$$

$$5) 4 - x^2 - 4y^2 \geq 0 \quad \left(\frac{x^2}{4} + y^2 \leq 1 \right) \text{ ve } y - x^2 > 0 \text{ olmalı}$$



$$6) \frac{\partial z}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} = \left(4xy^3 - \frac{1}{x-y} \right) \cdot r \cos(rt) + \left(6x^2y^2 + \frac{1}{x-y} \right) (-r \sin t)$$

$$\frac{\partial z}{\partial t} \Big|_{(r,t)=(1, \frac{\pi}{2})} = -1 //$$



2018-2019 BAHAR YARIYILI İNŞAAT MÜHENDİSLİĞİ BÖLÜMÜ Z.MAT1008 MATEMATİK II
ARASINAV SORULARIDIR. (08.04.2019)

- 1)
$$\begin{cases} -x + 2y + z + 5t = -1 \\ x + y + 2z + 7t = 4 \\ 3x + z + 7t = 5 \end{cases}$$
 lineer denklem sisteminin çözümünü elemanter satır (indirgeme) işlemleri yardımıyla bulunuz. (20 puan)

- 2) $A = \begin{bmatrix} \lambda & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 \\ -1 & 0 & 3 & 1 \\ 3 & 1 & 2 & 0 \end{bmatrix}$ matrisi verilsin. $\det A = 0$ olması için λ ne olmalıdır? (15 puan)

- 3) $P(1,2,-3)$ noktasından geçen ve $\vec{u} = \langle 1,0,3 \rangle = \vec{i} + 3\vec{k}$, $\vec{v} = \langle 4,1,2 \rangle = 4\vec{i} + \vec{j} + 2\vec{k}$ vektörlerine paralel olan düzlemin denklemini bulunuz. (15 puan)

- 4) $\vec{\alpha}(t) = \frac{4}{5} \cos t \vec{i} + (1 - \sin t) \vec{j} - \frac{3}{5} \cos t \vec{k}$ eğrisi için $\kappa(t) = \frac{\|\alpha'(t) \times \alpha''(t)\|}{\|\alpha'(t)\|^3} = ?$ ($= \frac{|\frac{d\alpha}{dt} \times \frac{d^2\alpha}{dt^2}|}{|\frac{d\alpha}{dt}|^3}$) (15 puan)

- 5) $f(x,y) = \sqrt{4 - x^2 - 4y^2} + \ln(y - x^2)$ şeklinde tanımlanan reel değerli fonksiyonun en geniş tanım kümesini gösteren bölgeyi xy düzlemi üzerinde gösteriniz. (20 puan)

- 6) $z = f(x,y) = 2x^2y^3 - \ln(x-y)$, $x = \sin(rt)$, $y = r \cos t$ için $\frac{\partial z}{\partial t}$ yi $(r,t) = (1, \frac{\pi}{2})$ noktasında hesaplayınız. (15 puan)

Not: Sınav süresi 90 dakikadır. İlk 30 dakika sınav salonunu terk etmek yasaktır.