CEVAP ANAHTARI (HARİTA MÜH. BÜTÜNLEME)

1) $\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 2 & -1 \\ 1 & -\lambda & 1 \\ 4 & -4 & 5-\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & 2-\lambda & 0 \\ 1 & -\lambda & 1 \\ 4 & -4 & 5-\lambda \end{vmatrix}$

$= (2-\lambda)\begin{vmatrix} 1 & 1 & 0 \\ 1 & -\lambda & 1 \\ 4 & -4 & 5-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & 1 & 0 \\ 0 & -1-\lambda & 1 \\ 0 & -8 & 5-\lambda \end{vmatrix}$

$= (2-\lambda)\left((-1-\lambda)(5-\lambda)+8\right) = (2-\lambda)(\lambda^2-5\lambda+\lambda-5+8)$

$= (2-\lambda)(\lambda^2-4\lambda+3) = (2-\lambda)(\lambda-3)(\lambda-1) = 0$

$\lambda_1 = 2, \; \lambda_2 = 3, \; \lambda_3 = 1$

$\lambda_2 = 3$ için;

$\begin{bmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$-2x_1 + 2x_2 - x_3 = 0$
$x_1 - 3x_2 + x_3 = 0$
$4x_1 - 4x_2 + 2x_3 = 0$

$\Rightarrow \; x_1 = -x_2, \; x_3 = 4x_2$

$\vec{U}_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_2 \\ 4x_2 \end{bmatrix} = x_2\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}, \; x_2 \in \mathbb{R}-\{0\}$

2) $f_x = \dfrac{x^2+y^2 - x2x}{(x^2+y^2)^2} = \dfrac{y^2-x^2}{(x^2+y^2)^2}, \quad f_x(1,2,\tfrac{1}{5}) = \dfrac{3}{25}$

$f_y = -\dfrac{2xy}{(x^2+y^2)^2}, \quad f_y(1,2,\tfrac{1}{5}) = -\dfrac{4}{25}, \quad f_z = -1$

$(D_{\vec{u}} f)(P) = \dfrac{3}{25}\cdot\dfrac{1}{3} + \left(-\dfrac{4}{25}\right)\left(-\dfrac{2}{3}\right) + (-1)\dfrac{2}{3} = -\dfrac{39}{75}$

3) $$\int_0^2\int_0^{x/2} e^{-y/x}\,dy\,dx = \int_0^2 \frac{e^{-y/x}}{-\frac{1}{x}}\Bigg|_0^{x/2} dx = \int_0^2 \left(-e^{-1/2}-1\right)x\,dx.$$

$$= \left(-e^{-1/2}-1\right)\frac{x^2}{2}\Bigg|_0^2 = -2\left(e^{-1/2}+1\right)$$

4) $g(x,y,z) = x^2+y^2+z^2-1$

$\nabla f = \lambda \nabla g \quad , \quad g(x,y,z)=0$

$\vec{i}+\vec{j}-\vec{k} = \lambda\left(2x\vec{i}+2y\vec{j}+2z\vec{k}\right)$

$x = \frac{1}{2\lambda} \;,\; y = \frac{1}{2\lambda} \;,\; z = -\frac{1}{2\lambda}$ ve $x^2+y^2+z^2=1$

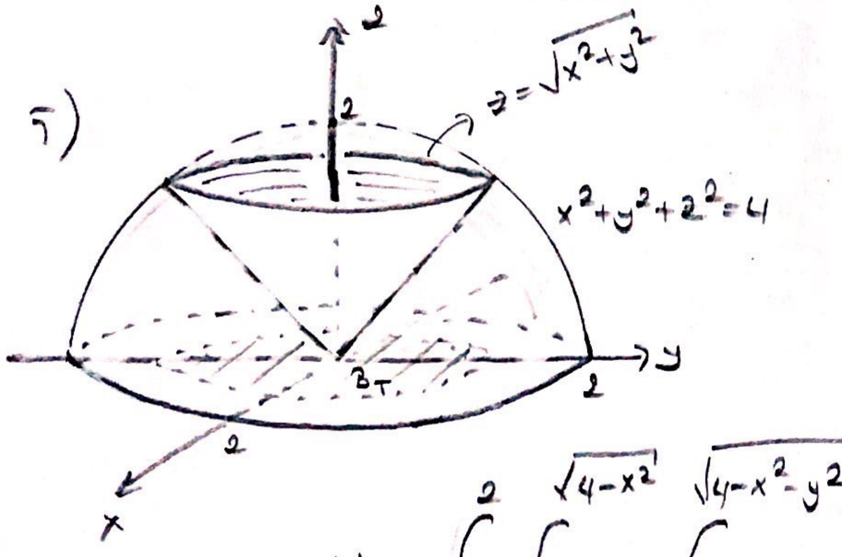
$\frac{1}{4\lambda^2}+\frac{1}{4\lambda^2}+\frac{1}{4\lambda^2} = 1 \Rightarrow \frac{3}{4\lambda^2}=1 \Rightarrow \lambda = \mp\frac{\sqrt{3}}{2}$

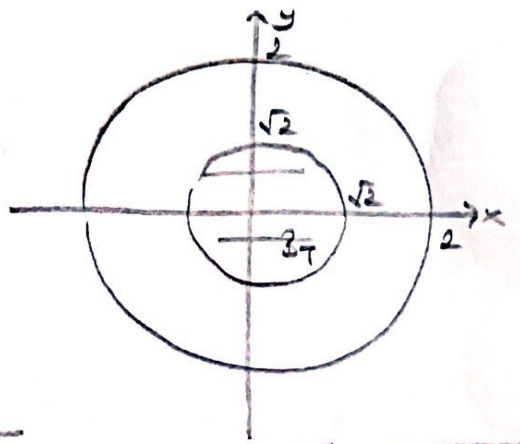
$\Rightarrow \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ ve $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) = \sqrt{3}$, $f\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = -\sqrt{3}$

$\downarrow$ maksimum $\qquad\qquad$ $\downarrow$ minimum

5)

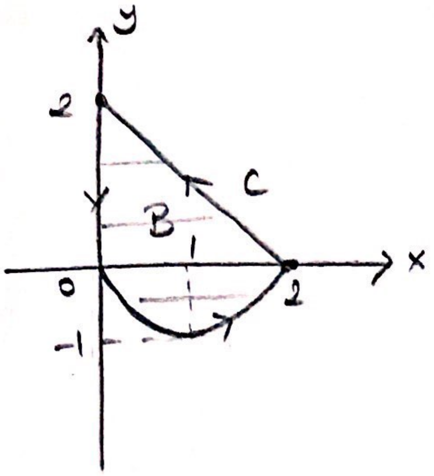




$$V = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} dz\,dy\,dx - \int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} dz\,dy\,dx$$

$$(\text{veya}) = \int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

6)



$$\oint_C \underbrace{(\arctan(x^2) + 3y^2)}_{P(x,y)} dx + \underbrace{(y\ln y + x^3)}_{Q(x,y)} dy = \iint_B (Q_x - P_y)\, dx\, dy$$

$$= \int_0^2 \int_{(x-1)^2-1}^{2-x} (3x^2 - 6y)\, dy\, dx$$

$$= \int_0^2 \left(3x^2y - 3y^2 \Big|_{(x-1)^2-1}^{2-x} \right) dx$$

$$= -2$$