

Ad-Soyad:

Dönem: 2022/2023 Güz

Numara:

Grup: A ☐ B ☐

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Bölüm: İnşaat Mühendisliği

* Her soru 20 puan! Süre 80 dk!

DİFERANSİYEL DENKLEMLER (MAT2011) ARASINAV SORULARI

1) $xy' - y = y^2 e^x$ diferansiyel denkleminin genel çözümünü bulunuz.

$$y' - \frac{1}{x}y = y^2 \frac{e^x}{x} \Rightarrow v = y^{-2} = y^{-1} \Rightarrow \frac{dv}{dx} = -y^{-2} \frac{dy}{dx} \quad (5)$$

$$\Rightarrow \frac{dy}{dx} = -y^2 \frac{dv}{dx} \Rightarrow -y^2 \frac{dv}{dx} - \frac{1}{x}y = y^2 \frac{e^x}{x}$$

$$\Rightarrow \frac{dv}{dx} + \frac{1}{x}v = -\frac{e^x}{x} \Rightarrow u(x) = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$\Rightarrow \frac{d}{dx}(x \cdot v) = -e^x \Rightarrow x \cdot v = -\int e^x dx$$

$$\Rightarrow v \cdot x = -e^x + C \Rightarrow v = -\frac{e^x}{x} + \frac{C}{x}$$

$$\Rightarrow y^{-1} = -\frac{e^x}{x} + \frac{C}{x} \quad (5)$$

2) $\frac{(1 + y^2 e^{xy})}{M} dx + \frac{(1 + xy)e^{xy}}{N} dy = 0$ diferansiyel denkleminin genel çözümünü bulunuz.

$$M_y = 2ye^{xy} + xy^2 e^{xy}, N_x = ye^{xy} + ye^{xy} + xy^2 e^{xy} \Rightarrow M_y = N_x \quad (5)$$

$$F_x = M \Rightarrow F = \int M dx = \int (1 + y^2 e^{xy}) dx = x + ye^{xy} + h(y) \quad (5)$$

$$F_y = N \Rightarrow e^{xy} + xy e^{xy} + h'(y) = e^{xy} + xy e^{xy} \quad (5)$$

$$\Rightarrow h'(y) = 0 \Rightarrow h(y) = C_1$$

$$\Rightarrow F = x + ye^{xy} = C \quad (5)$$

3) $(xy + y)dx + (xy + x)dy = 0, y(1) = 1$ başlangıç değer probleminin çözümünü bulunuz.

$$y(x+1)dx = -x(y+1)dy \Rightarrow \frac{x+1}{x} dx = -\frac{y+1}{y} dy \quad (5)$$

$$\Rightarrow \int (1 + \frac{1}{x}) dx = -\int (1 + \frac{1}{y}) dy \Rightarrow x + \ln|x| = -y - \ln|y| + C$$

$$\Rightarrow x + y + \ln|x \cdot y| = C \quad (5)$$

$$y(1) = 1 \Rightarrow 1 + 1 + \ln 1 = C \Rightarrow C = 2 \quad (5)$$

$$\Rightarrow x + y + \ln|x \cdot y| = 2 \quad (5)$$

4) $xy' = y - x \tan \frac{y}{x}$ diferansiyel denkleminin genel çözümünü bulunuz.

$$\Rightarrow y' = \frac{y}{x} - \tan \frac{y}{x} \Rightarrow f(tx, ty) = \frac{ty}{tx} - \tan \frac{ty}{tx} = \frac{y}{x} - \tan \frac{y}{x} = f(x, y) \quad (5)$$

$$\Rightarrow \text{Homogen!} \Rightarrow \frac{y}{x} = v \Rightarrow y = vx \Rightarrow y' = v + v'x$$

$$\Rightarrow v + v'x = \frac{vx}{x} - \tan \frac{vx}{x} = v - \tan v$$

$$\Rightarrow \frac{dv}{dx} x = -\tan v \Rightarrow \frac{dv}{\tan v} = -\frac{dx}{x} \quad (5)$$

$$\Rightarrow \int \frac{\cos v}{\sin v} dv = -\int \frac{dx}{x} \Rightarrow \ln |\sin v| = -\ln |x| + c \quad (5)$$

$$\Rightarrow \ln |x \sin v| = c \Rightarrow x \cdot \sin \frac{y}{x} = C, \quad C := \pm e^c. \quad (5)$$

5) $y'' - 3y' - 4y = e^{-x}$ diferansiyel denkleminin genel çözümünü bulunuz.

$$y'' - 3y' - 4y = 0 \Rightarrow \alpha^2 - 3\alpha - 4 = 0 \Rightarrow (\alpha - 4)(\alpha + 1) = 0$$

$$\Rightarrow \alpha_1 = 4, \alpha_2 = -1 \Rightarrow u = c_1 e^{4x} + c_2 e^{-x} \quad (5)$$

$$f(x) = e^{-x} \Rightarrow P(x) = 1, \alpha = -1 \Rightarrow k = 1, n = 0$$

$$\Rightarrow v = x e^{-x} A_0 \quad (5) \Rightarrow v' = A_0 e^{-x} - A_0 x e^{-x}$$

$$\Rightarrow v'' = -A_0 e^{-x} - A_0 e^{-x} + A_0 x e^{-x} = -2A_0 e^{-x} + A_0 x e^{-x}$$

$$\Rightarrow -2A_0 e^{-x} + A_0 x e^{-x} - 3A_0 e^{-x} + 3A_0 x e^{-x} - 4A_0 x e^{-x} = e^{-x}$$

$$-5A_0 = 1 \Rightarrow A_0 = -\frac{1}{5}$$

$$\Rightarrow v = -\frac{1}{5} x e^{-x} \quad (5)$$

$$\Rightarrow y = u + v = c_1 e^{4x} + c_2 e^{-x} - \frac{1}{5} x e^{-x} \quad (5)$$