2017-2018 BAHAR YARIYILI MÜHENDİSLİK FAKÜLTESİ ENDUSTRİ MÜHENDİSLİĞİ MÜHENDİSLİK MATEMATİĞİ (Z.MAT 2018) DERSİ ARASINAV SORULARIDIR. 09.04.2018

1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ olmak üzere $A^2 - 3A - 4I = ?$
Burada I, 2×2 tipinde birim matristir. **(15 puan)**

2) Bir A kare matrisi, $A^t = A^{-1}$ olması halinde ortogonal matris diye adlandırılır. Buna göre, $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & x & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ matrisi ortogonal matris ise $x = ?$
Burada A^t, A matrisinin transpozudur. **(15 puan)**

3) $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 0 & -3 \\ 0 & 4 & 1 \end{bmatrix}$ matrisinin (varsa) ters matrisini bulunuz. **(15 puan)**

4) $\left.\begin{array}{r} 2x_1 - 3x_2 + 5x_3 + 7x_4 = 1 \\ 4x_1 - 6x_2 + 2x_3 + 3x_4 = 2 \\ 2x_1 - 3x_2 + 11x_3 - 15x_4 = 1 \end{array}\right\}$
lineer denklem sisteminin çözümünü elemanter satır (indirgeme) işlemleri yardımıyla bulunuz. **(20 puan)**

5) a' nın alacağı değerlere göre,
$\left.\begin{array}{r} x_1 + (1+a)x_2 + (1+2a)x_3 + x_4 = 0 \\ 2x_1 + (4+2a)x_2 + (3+4a)x_3 + 6x_4 = 0 \\ -3x_1 - 3(1+a)x_2 - 6ax_3 + 3x_4 = 0 \\ -x_1 - (1+a)x_2 - (1+2a)x_3 - ax_4 = 0 \end{array}\right\}$
homojen lineer denklem sisteminin çözümlerini bulunuz. **(20 puan)**

6) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ matrisinin öz değerlerini ve bu öz değerlere karşılık gelen öz vektörlerini bulunuz. **(15 puan)**

Not: Sınav süresi 90 dakikadır. İlk 30 dakika sınav salonunu terk etmek yasaktır.

Z.MAT2018 ARASINAV CEVAP ANAHTARI (09.04.2018)

1) $A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 9 & 10 \end{bmatrix}$ 5P

$3A = \begin{bmatrix} 3 & 6 \\ 9 & 6 \end{bmatrix}$, $4I = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ 5P

$\Bigg\}\ A^2 - 3A - 4I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 5P

2) $A^t = A^{-1}$ ise $A.A^t = I$ dır.

$$\begin{bmatrix} 1/3 & 2/3 & -2/3 \\ -2/3 & x & -2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix}\begin{bmatrix} 1/3 & 2/3 & -2/3 \\ -2/3 & x & -2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix}^t = \begin{bmatrix} 1 & -2/9 + 2/3x + 4/9 & 0 \\ -2/9 + 2/3x + 4/9 & 4/9 + x^2 + 4/9 & 4/9 + 2/3x + 4/9 \\ 0 & 4/9 + 2/3x - 2/9 & 1 \end{bmatrix}$$ 5P

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ olmalıdır. 5P

$-\frac{2}{9} + \frac{2}{3}x + \frac{4}{9} = 0 \Rightarrow \frac{2}{3}x = -\frac{2}{9} \Rightarrow x = -\frac{1}{3}$ 5P

3) $\overset{2}{\curvearrowleft}\begin{vmatrix} 1 & -1 & 2 \\ -2 & 0 & -3 \\ 0 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 0 & -2 & 1 \\ 0 & 4 & 1 \end{vmatrix} = -6 \neq 0$ olduğundan A^{-1} vardır. 5P

$$A^{-1} = \frac{1}{-6}\begin{bmatrix} 12 & 2 & -8 \\ 9 & 1 & -4 \\ 3 & -1 & -2 \end{bmatrix}^t = -\frac{1}{6}\begin{bmatrix} 12 & 9 & 3 \\ 2 & 1 & -1 \\ -8 & -4 & -2 \end{bmatrix}$$ 5P

$$= \begin{bmatrix} -2 & -\frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{3} & -\frac{1}{6} & \frac{1}{6} \\ \frac{4}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$ 5P

4)

$$\begin{array}{cc} -1 & -2 \end{array}\left[\begin{array}{cccc|c} 2 & -3 & 5 & 7 & 1 \\ 4 & -6 & 2 & 3 & 2 \\ 2 & -3 & 11 & -15 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & -3 & 5 & 7 & 1 \\ 0 & 0 & -8 & -11 & 0 \\ 0 & 0 & 6 & -22 & 0 \end{array}\right] \begin{array}{c} \\ \\ \frac{1}{6} \end{array}$$

5P 5P

$$\begin{array}{cc} -5 & 8 \end{array} \sim \left[\begin{array}{cccc|c} 2 & -3 & 5 & 7 & 1 \\ 0 & 0 & -8 & -11 & 0 \\ 0 & 0 & 1 & -11/3 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & -3 & 0 & 76/3 & 1 \\ 0 & 0 & 0 & -122/3 & 0 \\ 0 & 0 & 1 & -11/3 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 2 & -3 & 0 & 76/3 & 1 \\ 0 & 0 & 1 & -11/3 & 0 \\ 0 & 0 & 0 & -122/3 & 0 \end{array}\right]$$

5P

$$\left.\begin{array}{l} 2x_1 - 3x_2 + 76/3\, x_4 = 1 \\ \left.\begin{array}{l} x_3 - 11/3\, x_4 = 0 \\ -\frac{122}{3} x_4 = 0 \Rightarrow x_4 = 0 \end{array}\right\} \Rightarrow x_3 = 0 \end{array}\right\} \begin{array}{l} 2x_1 = 1 + 3x_2 \\ x_1 = \frac{1}{2} + \frac{3}{2} x_2 \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1/2 + 3/2\, x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix}, \quad x_2 \in \mathbb{R}.$$

5P

5)

$$\begin{array}{ccc} 1 & 3 & -2 \end{array}\begin{bmatrix} 1 & (1+a) & (1+2a) & 1 \\ 2 & (4+2a) & (3+4a) & 6 \\ -3 & (-3-3a) & -6a & 3 \\ -1 & (-1-a) & (-1-2a) & -a \end{bmatrix} \sim \begin{bmatrix} 1 & (1+a) & (1+2a) & 1 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 1-a \end{bmatrix}$$

5P

$1-a \neq 0$ ise, yani $a \neq 1$ ise, sistemin çözümü $x_1 = x_2 = x_3 = x_4 = 0$.

5P

$1-a = 0$ ise, yani $a = 1$ ise,

$$\begin{array}{c} \\ -1 \\ -\frac{1}{3} \\ \\ \end{array}\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{array}{c} \\ \\ -1 \quad -2 \\ \\ \end{array}$$

✓5p

$$\sim \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{array}{r} x_1 - 7x_4 = 0 \\ 2x_2 + 2x_4 = 0 \\ x_3 + 2x_4 = 0 \end{array}\right\} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7x_4 \\ -x_4 \\ -2x_4 \\ x_4 \end{bmatrix}, \quad x_4 \in \mathbb{R}.$$

✓5p

6) $\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1) = 0$

$\lambda_1 = 4, \quad \lambda_2 = -1$ ✓5p

$\lambda_1 = 4$ için,

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \underset{2k}{\underset{\downarrow}{3x_1}} - \underset{3k}{\underset{\downarrow}{2x_2}} = 0 \qquad \vec{U}_1 = \begin{bmatrix} 2k \\ 3k \end{bmatrix} = k\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$k \in \mathbb{R} - \{0\}$ ✓5p

$\lambda_2 = -1$ için,

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 + x_2 = 0 \qquad \vec{U}_2 = \begin{bmatrix} -k \\ k \end{bmatrix} = k\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$k \in \mathbb{R} - \{0\}$

✓5p