CEVAP ANAHTARI (İNŞAAT MÜH. BÜTÜNLEME)

1) $$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 2 & -1 \\ 1 & -\lambda & 1 \\ 4 & -4 & 5-\lambda \end{vmatrix} = (2-\lambda)(\lambda-3)(\lambda-1)=0$$

$\lambda_1=2,\ \lambda_2=3,\ \lambda_3=1$

$\lambda_2=3$ için;

$$\begin{bmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{aligned} -2x_1+2x_2-x_3&=0 \\ x_1-3x_2+x_3&=0 \\ 4x_1-4x_2+2x_3&=0 \end{aligned}$$

$\Rightarrow x_1=-x_2,\ x_3=4x_2$

$$\vec{U}_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix},\quad x_2 \in \mathbb{R}-\{0\}$$

2) $$f(x,y,z) = \frac{x}{x^2+y^2} - z = 0$$

$$\nabla f = \frac{y^2-x^2}{(x^2+y^2)^2}\vec{i} + \left(-\frac{2xy}{(x^2+y^2)^2}\right)\vec{j} - \vec{k}$$

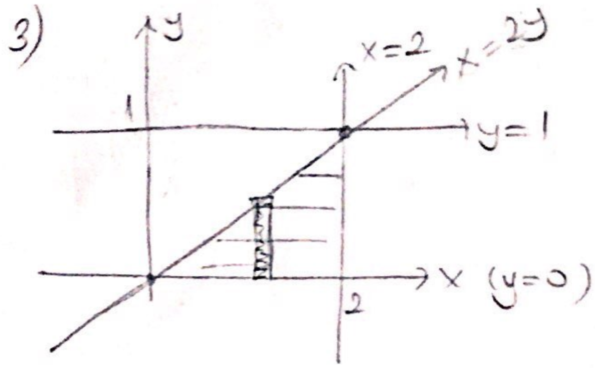
$$\vec{n} = \nabla f\big|_P = \frac{3}{25}\vec{i} - \frac{4}{25}\vec{j} - \vec{k}$$

$$\frac{3}{25}x - \frac{4}{25}y - z + D = 0 \qquad \text{P noktası düzlem üzerinde olacağından;}$$

$$\frac{3}{25}\cdot 1 - \frac{4}{25}\cdot 2 - \left(\frac{1}{5}\right) + D = 0 \Rightarrow D = \frac{10}{25}$$

$$\frac{3}{25}x - \frac{4}{25}y - z + \frac{10}{25} = 0 \quad \text{veya} \quad 3x-4y-25z+10=0$$

3)



$$\int_0^2 \int_0^{x/2} e^{-y/x}\, dy\, dx =$$

$$= \int_0^2 \frac{e^{-y/x}}{-\frac{1}{x}} \Big|_0^{x/2} dx = \int_0^2 (-e^{-1/2} - 1)x\, dx$$

$$= (-e^{-1/2}-1)\frac{x^2}{2}\Big|_0^2 = -2(e^{-1/2}+1)$$

4) $g(x,y,z) = x^2+y^2+z^2-1=0$

$\nabla f = \lambda \nabla g \quad , \quad g(x,y,z)=0$

$\vec{i} + \vec{j} - \vec{k} = \lambda(2x\vec{i} + 2y\vec{j} + 2z\vec{k})$

$x = \frac{1}{2\lambda} \quad , \quad y = \frac{1}{2\lambda} \quad , \quad z = -\frac{1}{2\lambda}$ ve $x^2+y^2+z^2=1$

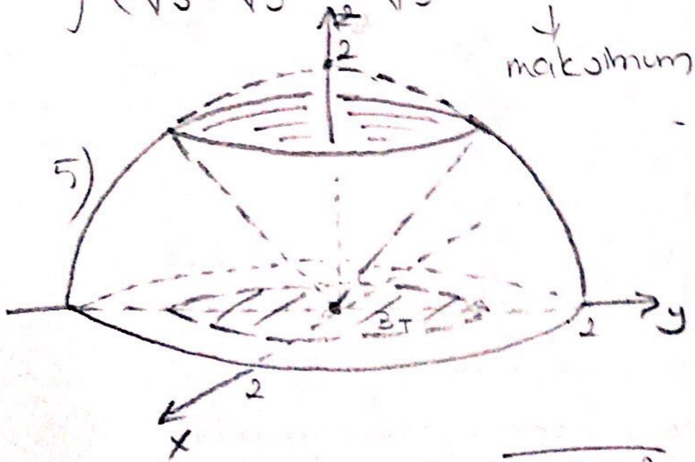
$\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 1 \Rightarrow \lambda = \mp \frac{\sqrt{3}}{2}$

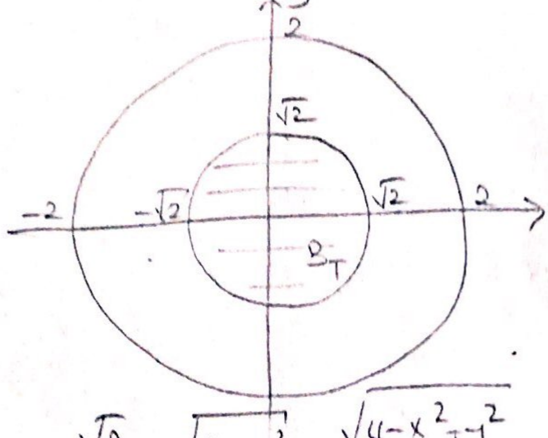
Buradan kritik noktalar $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ ve $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

$f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) = \sqrt{3}$ $\downarrow$ maksimum

$f\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = -\sqrt{3}$ $\downarrow$ minimum.

5)

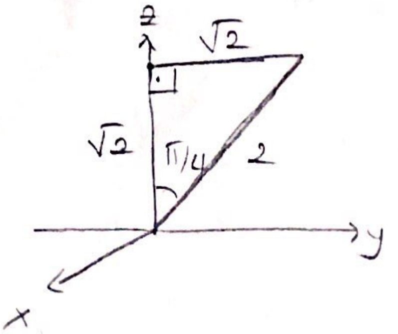




a) $$V = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} dz\, dy\, dx - \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} dz\, dy\, dx$$

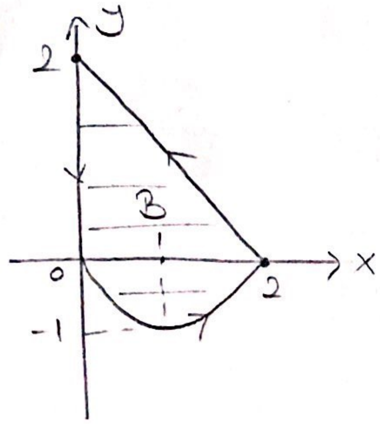
b) $z=\sqrt{x^2+y^2}$, $x^2+y^2+z^2=4$ ortak çözümü yapılırsa; "3))"

$z^2+z^2=4$ ve $z=\sqrt{2}$



$$V=\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2}\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

6)



$$\oint_C \underbrace{(\arctan(x^2)+3y^2)}_{P(x,y)}dx+\underbrace{(y\ln y+x^3)}_{Q(x,y)}dy=$$

$$=\iint_B (Q_x-P_y)\,dx\,dy$$

$$=\int_0^2\int_{(x-1)^2-1}^{2-x}(3x^2-6y)\,dy\,dx$$

$$=\int_0^2 (3x^2y-3y^2)\Big|_{(x-1)^2-1}^{2-x}dx$$

$$=-2$$