ON THE CONCEPT OF CENTROID FOR ELEMENT χ IN $\mathbb{R}$

HASAN KELEŞ

Karadeniz Technical University, Faculty of Sicience,Deparment of Mathematics, e-mail: hasankeles@ktu.edu.tr, ORCID ID: 0000-0001-8436-890X

ABSTRACT

This article is about the centroid of any real number. In mathematics, real numbers are the basic of the concept of measure. Real numbers are contributed a lot to the history of Humanity and pioneered the structure of technological developments. Real numbers are enabled the concept of numbers to be unified in the process and this expansion of numbers is achieved by adding new operations to it. Many studies are carried out on the resulting number system. Some of these studies are logarithmic calculations, functional operations, logic and base studies. Binary logic allowed for communication and digital structure. This situation is contributed to every part of life. Therefore, due to the contribution of numbers to life, there is a significant need for a change in number systems and bases. The topic base of numbers is gained importance in 2023. The definition of centroid, which is related to the base, is given in 2024. The connection of numbers with the centroid is examined in this study. New situations related to numbers are investigated. The common aspect of sets and centralizers is discussed. The new perspective on numbers is gained with this aspect. In particular, different expressions of numbers are obtained. This situation is drew attention to the idea of numbers intertwined with logic. Centroids of positive integers are written. This centroid relation of real numbers is given. The course of numbers is analyzed. The foundation of logic with the centered is built. New developments of the centrist in itself are observed. The subject is researched in depth in order to pioneer further studies.

Keywords : base, centroid, logic, sayılar, bases numbers.

1. INTRODUCTION AND THE CONCEPT OF CENTROID FOR ELEMENT χ IN $\mathbb{R}$

The Mayans used a vigesimal (base 20) number system, the Babylonians used a sexagesimal (base 60) number system, and the Egyptians used a duo-decimal (base 12) number system.

The interest and need for numbers are increased with the concept of measurement in history. This situation is made mandatory to change the base with technology. Ithis study, the definition of centricity is focused on the prominent one in base changes.

Any number written in binary base can also be written in binary base B3. For example,

Number	Base B_2	In B_3
	Base 2	
2	10	1-10
3	11	1-11

Let $a > 1$ be an integer and $c_0, c_1, \ldots, c_n \in \{0, \ldots, a-1\}$ for $n \in \mathbb{Z}^+$. Any $k \in \mathbb{Z}$ is written uniquely in the form

$$k = \sum_{i=1}^{n} c_i a^i, \text{ where } n \in \mathbb{Z}^+.$$

Here a is the base and $c_0, c_1, \ldots, c_n$ are the digits.

Definition 1.1. The integer k is called a centroid with coefficient b in base a if $k = ba^s$ for constant integers $a, b,$ where $s \in \mathbb{Z}^+$.

Here the integer a is called the base of the centroid and the integer s is called the period $p(a)$ of the centroid. The period of elements with the same number of digits is considered as the positive integer s. The set of these elements is called the centroid set and is denoted by $C_{B_a}(k)$. The number of elements in $C_{B_a}(k)$ is denoted by $|C_{B_a}(k)|$. The number of coefficients of integer k written in base a is denoted by $d_{B_a}(k)$. With this notation, we have

$$C_{B_a}(k) = \{c \in \mathbb{Z}^+ | d_{B_a}(k) = d_{B_a}(c)\}.$$

Proposition 1.2. Let $x \in C_{B_a}(k)$ be a centroid. If $x \in C_{B_a}(k)$ then

$$c_0, c_1, \ldots, c_n \in \{0, 1\}.$$

Proof. If $x \in C_{B_a}(k)$ then

$$x = \sum_{i=0}^{n} c_i a^i, \text{ where } n \in \mathbb{Z}^+.$$

$$x = (c_n \ldots c_1 c_0)_r,$$

For $0 \leq i \leq n-1$, $c_i = 0$ and $c_n = 1$. Because,

$$x = a^n.$$

Example 1.2. In base B_2,

$$C_{B_2}(2) = C_{B_2}(3) \text{ and } C_{B_2}(4) = C_{B_2}(5) = C_{B_2}(6) = C_{B_2}(7).$$

$$C_{B_2}(2)=\{2,3\},\ C_{B_2}(4)=\{4,5,6,7\}\ .$$

Example 1.3. In base B_3,

$$C_{B_3}(2)=C_{B_3}(3)=C_{B_3}(4)\ \text{and}\ C_{B_3}(5)=C_{B_3}(6)=C_{B_3}(7)=\ =C_{B_3}(8)$$

$$=C_{B_3}(9)=C_{B_3}(10)=C_{B_3}(11).$$

$$C_{B_2}(2)=\{2,3\},\ C_{B_2}(4)=\{4,5,6,7\}\ .$$

Proposition 1.4. Let $C_{B_a}(k)$ be a centroid and $x \in C_{B_a}(k)$. If $x = a^s$ then the number of digits of x is s.

In the centroid $C_{B_a}(k)$, only one single element is written as $x = a^s$.

i. If $a=2$ and $b=1$ then $C_{B_2}(k)$ and c_0, $c_1, \dots, c_n \in \{0,\ 1\}$.

Number	Base in B_2	c	$C_{B_2}(k)$	$p(2)=s$	$d_{B_2}(c)$
1	1	2^0	1	0	1
2	10	2^1	2, 3	1	2
3	11				
4	100	2^2	4, 5, 6, 7	2	3
5	101				
6	110				
7	111				
8	1000	2^3	8, 9, 10, 11, 12, 13, 14, 15	3	4
9	1001				
10	1010				
11	1011				
12	1100				
13	1101				
14	1110				
15	1111				
16	10000	2^4	16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30,31	4	16
$\vdots$	$\vdots$				
31	11111				
32	100000	2^5	32, …, 63	5	32
$\vdots$	$\vdots$				
63	111111				

Proposition 1.5. Let $C_{B_2}(k)$ be a centroid. If $p(2)=s$, then the following hold.

i. $\min C_{B_2}\left(2^s\right)=2^{s+1}$.

ii. $\max C_{B_2}\left(2^s\right)=\sum_{i=0}^{s+1}2^i$.

iii. $\inf C_{B_2}\left(2^s\right)=\sum_{i=0}^{s}2^i$.

iv. $\sup C_{B_2}\left(2^s\right)=2^{s+1}$.

v. $\left|C_{B_2}\left(2^s\right)\right|=2^s$.

Proof. Let $C_{B_2}(k)$ be a centroid. If $p(2)=s$ and $x\in C_{B_2}(k)$ then,

i. $x=\left(\underset{(s+1)-times}{1\ldots}\right)_2$.

The smallest of these is $(10\ldots0)_2$.

$\min C_{B_2}\left(2^s\right)=2^{s+1}$

ii. The maximum of these is $(11\ldots1)_2$. $\max C_{B_2}\left(2^s\right)=\sum_{i=0}^{s+1}2^i$.

The proofs of iii and iv are clear.

The following theorem is given without proof in [3].

Theorem 1.6. Let $a>1$ be an integer and $c_0, c_1, \ldots, c_n \in\{-1,0,1\}$ for $n\in\mathbb{Z}^+$. Any $k\in\mathbb{Z}$ is written uniquely in the form

$$k=\sum_{i=0}^{n}c_i a^i, \text{ where } n\in\mathbb{Z}^+.$$

If any number is written using this theorem, this number is called to be written in base B_3. It is denoted by

$$k=(\ldots)_{B_3}.$$

The values and complements of some numbers are given in base B_2 and base B_3 at in the table below.

In our study, $a=3$ and $b=1$ are taken since the ternary number system is used in [3].

ii. If $a=3$ and $b=1$ then $C_{B_3}(k)$.

Number	Base in B_3		$C_{B_3}(k)$	$p(3)=s$	$d_{B_3}(3)$
1	1	3^0	1	0	1
2	1-1				
3	10	3^1	2, 3, 4	1	3
4	11				
5	1-1-1				
6	1-10				
7	1-11	3^2	5, 6, 7, 8, 9, 10, 11, 12, 13	2	9
8	10-1				
9	100				
10	101				
11	11-1				
12	110				
13	111				
14	1-1-1-1				
$\vdots$	$\vdots$	3^3	14, 15,…, 40	3	27
40	11111				
41	1-1-1-1-1				
$\vdots$	$\vdots$	3^4	41, …, 121	4	81
121	11111				
122	1-1-1-1-1-1				
$\vdots$	$\vdots$	3^5	122, …, 364	5	243
364	111111				

Proposition 1.7. Let $C_{B_2}(k)$ be a centroid. If $p(2)=s$, then the following hold.

i. $\min C_{B_3}(3^s)=3^{s+1}-\sum_{i=0}^{s}3^i$.

ii. $\max C_{B_3}(3^s)=\sum_{i=0}^{s+1}3^i$.

iii. $\inf C_{B_3}(3^s)=\sum_{i=0}^{s}3^i$.

iv. $\sup C_{B_3}(3^s)=3^{s+1}-\sum_{i=0}^{s}3^i$.

v. $\left|C_{B_3}\left(3^s\right)\right| = 3^s$.

Proof. The proof of the proposition is clear.

Let us give the following theorem without proof

Theorem 1.8. Let $a > 1$ be an integer. If $C_{B_a}(k)$ and $p(a) = s$ then

i. $\min C_{B_a}\left(a^s\right) = a^{s+1} - \sum_{i=0}^{s} a^i$.

ii. $\max C_{B_a}\left(a^s\right) = \sum_{i=0}^{s+1} a^i$.

iii. $\inf C_{B_a}\left(a^s\right) = \sum_{i=0}^{s} a^i$.

iv. $\sup C_{B_a}\left(a^s\right) = a^{s+1} - \sum_{i=0}^{s} a^i$.

v. $\left|C_{B_a}\left(a^s\right)\right| = a^s$.

3. RESULTS AND CONCLUSIONS

The definition of centred by triple logic is given. The expression of a new concept would also provide new studies.

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