Kuvvet Serileri:

$\{a_n\}$ bir dizi $a \in \mathbb{R}$, $x \in \mathbb{R}$ bir değişken olmak üzere

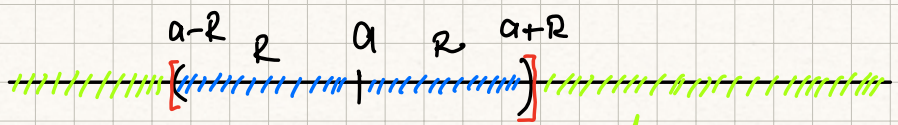
$$\sum_{n=0}^{+\infty} a_n (x-a)^n = a_0 + a_1(x-a) + a_2(x-a)^2 + \cdots + a_n(x-a)^n + \cdots$$

serisine bir kuvvet serisi denir.

$x=3 \Rightarrow \sum_{n=0}^{+\infty} a_n (3-a)^n$ bir reel sayı serisidir.

$x=-2 \Rightarrow \sum_{n=0}^{+\infty} a_n (-2-a)^n$ " " " " .

$x=a \Rightarrow \sum_{n=0}^{+\infty} a_n (a-a)^n$ " " " " .



$Y.M = a$

$Y.Y = R$

$Y.A = (a-R, a+R)$

$[\ ,\ ]$

$R=0 \Rightarrow$ $Y.M = a$, $Y.Y = 0$, $Y.A = \{a\}$

$R=+\infty$ $Y.M = a$, $Y.Y = +\infty$, $Y.A = (-\infty, +\infty)$

Örnekler:

1) $\sum_{n=0}^{+\infty} 2^n (x-1)^n$, $Y.M.$ $Y.Y.$ $Y.A.$ = ?

$$\lim_{n\to+\infty} \left| \frac{2^{n+1}(x-1)^{n+1}}{2^n (x-1)^n} \right| = \lim_{n\to+\infty} |2(x-1)| = \lim_{n\to+\infty} 2|x-1| = 2|x-1|$$

$\Rightarrow 2|x-1| < 1 \Rightarrow$ yak.

$\Rightarrow |x-1| < \frac{1}{2} \Rightarrow$ yok.

$\Rightarrow -\frac{1}{2} < x-1 < \frac{1}{2} \Rightarrow$ yok.

$\Rightarrow \frac{1}{2} < x < \frac{3}{2} \Rightarrow$ yok.

$\frac{1}{2}$ ($\frac{1}{2}$, 1 , $\frac{3}{2}$)

$x=\frac{1}{2} \Rightarrow \sum_{n=0}^{+\infty} 2^n \left(\frac{1}{2}-1\right)^n = \sum_{n=1}^{+\infty} 2^n \cdot \left(\frac{-1}{2}\right)^n = \sum_{n=1}^{+\infty} \frac{2^n (-1)^n}{2^n} = \sum_{n=1}^{+\infty} (-1)^n$

$\lim_{n\to+\infty} (-1)^n = \lim$ yok $\neq 0$

G.T.T. ıraksaktır.

$x=\frac{3}{2} \Rightarrow \sum_{n=0}^{+\infty} 2^n \left(\frac{1}{2}-1\right)^n = \sum_{n=0}^{+\infty} 2^n \cdot \frac{1}{2^n} = \sum_{n=0}^{+\infty} 1$

$\lim_{n\to+\infty} 1 = 1 \neq 0$

G.T.T. ıraksak.

Y.M = 1

Y.Y = $\frac{1}{2}$

Y.A = $\left(\frac{1}{2}, \frac{3}{2}\right)$

2) $\sum_{n=1}^{+\infty} (-1)^n \frac{e^n}{n!} (x+3)^n$

$$\lim_{n\to+\infty} \left| \frac{(-1)^{n+1} \frac{e^{n+1}}{(n+1)!} (x+3)^{n+1}}{(-1)^n \frac{e^n}{n!} (x+3)^n} \right| = \lim_{n\to+\infty} \left| (-1) \cdot e \frac{n!}{(n+1)!} (x+3) \right|$$

$(n+1)! = (n+1)!$

$= \lim_{n\to+\infty} e \frac{1}{n+1} |x+3| = 0 < 1 \Rightarrow$

Y.M = -3

Y.Y = $+\infty$

Y.A = $(-\infty, +\infty)$

3) $\sum_{n=0}^{+\infty} \frac{n!}{\ln n} x^n$

$$\lim_{n\to+\infty} \left| \frac{\frac{(n+1)!}{\ln(n+1)} x^{n+1}}{\frac{n!}{\ln n} x^n} \right| = \lim_{n\to+\infty} \left| \frac{(n+1)!}{n!} \frac{\ln n}{\ln(n+1)} x \right|$$

$$= \lim_{n\to+\infty} (n+1) \frac{\ln n}{\ln(n+1)} |x| = \begin{cases} 0, & x=0 \text{ ise} \\ +\infty, & x\neq 0 \text{ ise} \end{cases} < 1$$

($(n+1) \to +\infty$, $\frac{\ln n}{\ln(n+1)} \to 1$, $|x| \to$ sayı)

$Y.M = 0$

$Y.Y = 0$

$Y.A = \{0\}$

4) $\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n^2} \left(\frac{x-2}{3}\right)^n$

$$\lim_{n\to+\infty} \left| \frac{\frac{(-1)^{n+2}}{(n+1)^2} \left(\frac{x-2}{3}\right)^{n+1}}{\frac{(-1)^{n+1}}{n^2} \left(\frac{x-2}{3}\right)^n} \right| = \lim_{n\to+\infty} \left| (-1) \frac{n^2}{(n+1)^2} \cdot \frac{x-2}{3} \right|$$

$$= \lim_{n\to+\infty} \frac{n^2}{(n+1)^2} \frac{|x-2|}{3} = \frac{|x-2|}{3} < 1$$

$\Rightarrow |x-2| < 3$

$\Rightarrow -3 < x-2 < 3$

$\Rightarrow -1 < x < 5$

$Y.M = 2$

$Y.Y = 3$

$Y.A = [-1, 5]$

$x=-1 \Rightarrow \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n^2}\underbrace{\left(\frac{-1-2}{3}\right)^n}_{-1} = \sum_{n=1}^{+\infty} \frac{(-1)^{2n+1}}{n^2} = \sum_{n=1}^{+\infty} \frac{-1}{n^2} = -\sum_{n=1}^{+\infty} \frac{1}{n^2}$

p-Testi yakınsak.

$x=5 \Rightarrow \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n^2}\underbrace{\left(\frac{5-2}{3}\right)^n}_{1} = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n^2}$

Leibnitz Testi yakınsak.

5) $\sum_{n=2}^{+\infty} \frac{1.3.5.\cdots.(2n-3)}{3^n\, n!}\,(x-1)^n \qquad (-\tfrac{1}{2})^n$

$$\lim_{n\to+\infty} \left| \frac{\frac{1.3.5\cdots(2n-3).(2n-1)}{3^{n+1}(n+1)!}(x-1)^{n+1}}{\frac{1.3.5\cdots(2n-3)}{3^n\, n!}(x-1)^n} \right| = \lim_{n\to+\infty} \left| \frac{(2n-1)}{3}\frac{n!}{(n+1)!}(x-1) \right|$$

$(n+1)$

$$= \lim_{n\to+\infty} \frac{2n-1}{3(n+1)}|x-1| = \frac{2}{3}|x-1| < 1$$

$\Rightarrow |x-1| < \frac{3}{2}$

$\Rightarrow -\frac{3}{2} < x-1 < \frac{3}{2}$

$\Rightarrow -\frac{1}{2} < x < \frac{5}{2}$

Y.M = 1

Y.Y = $\frac{3}{2}$

Y.A = $(-\frac{1}{2}, \frac{5}{2})$

Uç. noktalar zor.

6) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{4^n}(x-2)^{2n+1}$

$$\lim_{n\to+\infty}\left|\frac{\frac{(-1)^{n+1}}{q^{n+1}}(x-2)^{2n+3}}{\frac{(-1)^n}{q^n}(x-2)^{2n+1}}\right| = \lim_{n\to+\infty}\left|\frac{(-1)}{q}(x-2)^2\right|$$

$$= \lim_{n\to+\infty}\frac{1}{q}|x-2|^2 = \frac{1}{q}|x-2|^2 < 1$$

$$\Rightarrow |x-2|^2 < q$$

$$\Rightarrow |x-2| < 3$$

$$\Rightarrow -3 < x-2 < 3$$

$$\Rightarrow -1 < x < 5$$

Y.M = 2

Y.Y = 3

Y.A = (-1, 5)

$$x=-1 \Rightarrow \sum_{n=1}^{+\infty}\frac{(-1)^n}{q^n}(-1-2)^{2n+1} = \sum_{n=1}^{+\infty}\frac{(-1)^n}{q^n}(-1)^{2n+1}\cdot 3^{2n+1} = \sum_{n=1}^{+\infty} 3$$

$$\lim_{n\to+\infty} 3 = 3 \neq 0$$

G.T.T. irahsalı

$$x=5 \Rightarrow \sum_{n=1}^{+\infty}\frac{(-1)^n}{q^n}(5-2)^{2n+1} = \sum_{n=1}^{+\infty}(-1)^n\cdot 3$$

G. T. T. iraksak

$$\lim_{n\to+\infty}(-1)^n\cdot 3 \neq 0$$

Alıştırmalar:

Aşağıdaki kuvvet serilerinin Y.M., Y.Y., Y.A. ?

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n} x^n \quad , \quad \sum_{n=1}^{+\infty} \frac{5^n}{n!} (x+2)^{n+2} \quad , \quad \sum_{n=1}^{+\infty} \frac{(4x-5)^n}{3^n}$$

$$\sum_{n=2}^{+\infty} \frac{(-1)^n (x-1)^n}{n \ln n} \quad , \quad \sum_{n=1}^{+\infty} \frac{3^n}{(-2)^n n(n+1)} (x+5)^n \quad , \quad \sum_{n=1}^{+\infty} \frac{n^2}{3^{2n}} (x+7)^{3n}$$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{2^n} (x-2)^{\frac{n}{3}} \quad , \quad \sum_{n=0}^{+\infty} \frac{5^n}{(2n)!} \left(\frac{x+1}{3}\right)^n \quad , \quad \sum_{n=1}^{+\infty} \left(\frac{n}{n+1}\right)^{n^2} (x-e)^n$$

$$\sum_{n=0}^{+\infty} \left(\frac{x^2+2}{6}\right)^{n^2}$$