2) $f(x)=\dfrac{3}{4x-7}$ fonksiyonunun $x_0=3$ nda. Taylor serisi?

$$f(x)=\frac{3}{4x-7}=3.(4x-7)^{-1}$$

$$f^{(1)}(x)=3.4.(-1).(4x-7)^{-2}$$

$$f^{(2)}(x)=3.4.(-1).4.(-2).(4x-7)^{-3}$$

$$f^{(3)}(x)=3.4.(-1).4.(-2).4.(-3)(4x-7)^{-4}$$

$$\vdots$$

$$f^{(n)}(x)=3.4^n(-1)^n n!(4x-7)^{-(n+1)} \qquad n=0,1,2,\dots,n,\dots$$

$$f^{(n)}(3)=3.4^n(-1)^n n!\,5^{-(n+1)}$$

$$\sum_{n=0}^{+\infty}\frac{f^{(n)}(3)}{n!}(x-3)^n=\sum_{n=0}^{+\infty}\frac{3\,4^n(-1)^n\cancel{n!}}{5^{n+1}\cancel{n!}}(x-3)^n$$

$$=\sum_{n=0}^{+\infty}\frac{3}{5}\left(\frac{-4}{5}\right)^n(x-3)^n$$

3) $y=7\ln\left(\sqrt[3]{\dfrac{8}{3x+1}}\right)$ fonksiyonunun $x_0=1$ Taylor serisi ve Maclaurin serisi?

$$y=7.\ln\left(\sqrt[3]{\frac{8}{3x+1}}\right)=7.\ln\left(\frac{\sqrt[3]{8}}{\sqrt[3]{3x+1}}\right)$$

$$=7.\ln\left(\frac{2}{(3x+1)^{1/3}}\right)=7\ln\left(2.(3x+1)^{-1/3}\right)$$

$$=7\ln\left(\left(\frac{3x+1}{8}\right)^{-1/3}\right)=-\frac{7}{3}\ln\left(\frac{3}{8}x+\frac{1}{8}\right)$$

$$f(x)=-\frac{7}{3}\ln\left(\frac{3}{8}x+\frac{1}{8}\right)$$

$$f^{(1)}(x)=-\frac{7}{3}.\frac{3}{8}.\frac{1}{\frac{3}{8}x+\frac{1}{8}}=-\frac{7}{3}.\frac{3}{8}.\left(\frac{3}{8}x+\frac{1}{8}\right)^{-1}$$

$$f^{(2)}(x) = -\frac{7}{3}\cdot\frac{3}{8}\cdot\frac{3}{8}\cdot(-1)\cdot\left(\frac{3}{8}x+\frac{1}{8}\right)^{-2}$$

$$f^{(3)}(x) = -\frac{7}{3}\cdot\frac{3}{8}\cdot\frac{3}{8}\cdot(-1)\cdot\frac{3}{8}\cdot(-2)\cdot\left(\frac{3}{8}x+\frac{1}{8}\right)^{-3}$$

$$f^{(4)}(x) = -\frac{7}{3}\cdot\frac{3}{8}\cdot\frac{3}{8}\cdot(-1)\,\frac{3}{8}\cdot(-2)\cdot\frac{3}{8}\cdot(-3)\cdot\left(\frac{3}{8}x+\frac{1}{8}\right)^{-4}$$

$$\vdots$$

$$f^{(n)}(x) = \left(-\frac{7}{3}\right)\cdot\left(\frac{3}{8}\right)^{n}\cdot(-1)^{n-1}(n-1)!\left(\frac{3}{8}x+\frac{1}{8}\right)^{-n}, \quad n=1,2,3,\dots$$

$$\sum_{n=0}^{+\infty}\frac{f^{(n)}(1)}{n!}(x-1)^{n} = \frac{f^{(0)}(1)}{0!}(x-1)^{0}+\sum_{n=1}^{+\infty}\frac{f^{(n)}(1)}{n!}(x-1)^{n}$$

$$= \frac{-\frac{7}{3}\ln\left(\frac{1}{2}\right)}{1}\cdot 1+\sum_{n=1}^{+\infty}\frac{\left(-\frac{7}{3}\right)\left(\frac{3}{8}\right)^{n}\cdot(-1)^{n-1}\cancel{(n-1)!}\left(\frac{1}{2}\right)^{-n}}{\cancel{n!}\; n}(x-1)^{n}$$

$$= \ln\left(\sqrt[3]{128}\right)+\sum_{n=1}^{+\infty}\frac{\frac{7}{3}\left(-\frac{3}{4}\right)^{n}}{n}(x-1)^{n}$$

$$\sum_{n=0}^{+\infty}\frac{f^{(n)}(0)}{n!}x^{n} = f(0)+\sum_{n=1}^{+\infty}\frac{f^{(n)}(0)}{n!}x^{n}$$

$$= -\frac{7}{3}\ln\left(\frac{1}{8}\right)+\sum_{n=1}^{+\infty}\frac{-\frac{7}{3}\left(\frac{3}{\cancel{8}}\right)^{n}\cdot(-1)^{n-1}\cancel{(n-1)!}\,\cancel{8^{n}}}{\cancel{n!}\; n}x^{n}$$

$$= \ln(128)+\sum_{n=1}^{+\infty}\frac{7}{3}\,\frac{(-3)^{n}}{n}x^{n}$$

4) $f(x)=\sqrt{5x+3}$, $x=2$ taylor? Maclaurin?

$$f(x)=(5x+3)^{1/2}$$

$$f^{(1)}(x)=5.\frac{1}{2}(5x+3)^{-\frac{1}{2}}$$

$$f^{(2)}(x)=5.\frac{1}{2}.5.\left(-\frac{1}{2}\right)(5x+3)^{-\frac{3}{2}}$$

$$f^{(3)}(x)=5.\frac{1}{2}.5.\left(-\frac{1}{2}\right).5.\left(-\frac{3}{2}\right)(5x+3)^{-\frac{5}{2}}$$

$$f^{(4)}(x)=5.\frac{1}{2}.5.\left(-\frac{1}{2}\right).5.\left(-\frac{3}{2}\right).5.\left(-\frac{5}{2}\right)(5x+3)^{-\frac{7}{2}}$$

$$\vdots$$

$$f^{(n)}(x)=5^n\frac{1}{2^n}(-1)^n.1.3.5\ldots(2n-3)(5x+3)^{-\frac{(2n-1)}{2}},$$

$n=2,3,4,\ldots$

$$\sum_{n=0}^{+\infty}\frac{f^{(n)}(2)}{n!}(x-2)^n=\overset{\sqrt{13}}{f(2)}+\overset{\frac{5}{2\sqrt{13}}}{f^{(1)}(2)}(x-2)+$$

$$+\sum_{n=2}^{+\infty}\frac{5^n\frac{1}{2^n}(-1)^n\,1.3.5\ldots(2n-3)\;13^{-\frac{(2n-1)}{2}}}{n!}(x-2)^n$$

$$\sum_{n=0}^{+\infty}\frac{f^{(n)}(0)}{n!}x^n=\overset{\sqrt{3}}{f(0)}+\overset{5/2\sqrt{3}}{f^{(1)}(0)}\,x+$$

$$+\sum_{n=2}^{+\infty}\frac{5^n\frac{1}{2^n}(-1)^n.\,1.3.5\ldots(2n-3)\;3^{-\frac{(2n-1)}{2}}}{n!}x^n$$

Alıştırmalar:

1) $f(x) = \frac{3}{x+3}$, $x_0 = 2$ Taylor?

2) $f(x) = \sqrt{e^{3x-1}}$, $x_0 = 4$ " ? Maclaurin?

3) $f(x) = 3\ln(4x-12)$, $x_0 = 5$ Taylor?

4) $f(x) = \sqrt{3-2x}$, $x_0 = 1$ Taylor?

5) $f(x) = \frac{4x+1}{(3x-7)(2x+1)}$, $x_0 = 2$ Taylor?

6) $f(x) = \frac{3}{(2x+3)^3}$, $x_0 = 1$ Taylor?