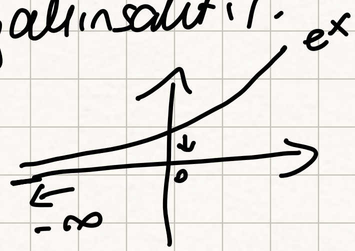
5) $\left\{ \ln\left(\frac{n+2}{n+1}\right) \right\}$ yakınsak mıdır?

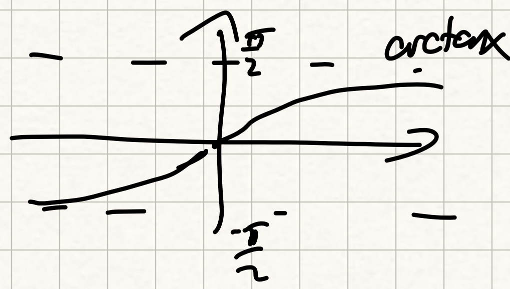
$$\lim_{n\to+\infty} \ln\left(\frac{n+2}{n+1}\right) = \ln\left(\lim_{n\to+\infty} \underbrace{\frac{n+2}{n+1}}_{\frac{\infty}{\infty}}\right) = \ln\left(\lim_{n\to+\infty} \frac{1}{1}\right)$$

$= \ln(1) = 0$ olup $\left\{ \ln\left(\frac{n+2}{n+1}\right) \right\}$ yakınsaktır.

6) $\left\{ \frac{\sin\left(\frac{1}{n}\right) + 2e^{-n}}{\frac{\pi}{2} - \arctan n} \right\}$ yakınsak mıdır.



$$\lim_{n\to+\infty} \frac{\sin\left(\frac{1}{n}\right) + 2e^{-n}}{\frac{\pi}{2} - \arctan n} \overset{\frac{0}{0}}{=}$$



$$= \lim_{n\to+\infty} \frac{\cancel{-}\frac{1}{n^2} \cdot \cos\left(\frac{1}{n}\right) + 2(\cancel{-1}) \cdot e^{-n}}{\cancel{-}\frac{1}{1+n^2}}$$

$$= \lim_{n\to+\infty} \underbrace{\frac{1+n^2}{n^2}}_{1} \cdot \underbrace{\cos\left(\frac{1}{n}\right)}_{1} + 2 \cdot \underbrace{\frac{(1+n^2)}{e^n}}_{} = 1.1 + 2.0 = 1$$

yakınsaktır.

7) $\left\{ \left(n + \frac{2}{n}\right)^{\frac{1}{\sqrt{n}}} \right\}$ yakınsak mıdır.

$$\lim_{n\to+\infty} \left(n+\frac{2}{n}\right)^{\frac{1}{\sqrt{n}}} \overset{\infty^0}{=} e^0 = 1$$ yakınsaktır. Çünkü

$$\lim_{n\to+\infty} \frac{1}{\sqrt{n}} \cdot \ln\left(n+\frac{2}{n}\right) \overset{0.\infty}{=} \lim_{n\to+\infty} \frac{\ln\left(n+\frac{2}{n}\right)}{\sqrt{n}}$$

$$\frac{\infty}{\infty} \lim_{n\to+\infty} \frac{\left(1-\frac{2}{n^2}\right)\cdot \frac{1}{n+\frac{2}{n}}}{\frac{1}{2\sqrt{n}}} = \lim_{n\to+\infty} \frac{\frac{n^2-2}{\cancel{n^2}} \cdot \frac{\cancel{n}}{n^2+2}}{\frac{1}{2\sqrt{n}}}$$

$$= \lim_{n\to+\infty} \underbrace{\frac{n^2-2}{n^2+2}}_{1} \cdot \frac{2\cancel{\sqrt{n}}}{\cancel{n}_{\sqrt{n}}} = \lim_{n\to+\infty} 1\cdot \frac{2}{\sqrt{n}} = 0$$

Teorem: Sonlu terim hariç geriye kalan sonsuz terimde $a_n \le b_n \le c_n$ eşitsizliği sağlansın. $\lim\limits_{n\to+\infty} a_n = L = \lim\limits_{n\to+\infty} c_n$ ise $\lim\limits_{n\to+\infty} b_n = L$'dir.

8) $\left\{(-1)^n \cdot \frac{\sin(n)}{\sqrt{n}}\right\}$ yakınsak mıdır.

$\forall n \in \mathbb{N}$ $\quad -1 \le \sin n \le 1$

$$-1 \le -\sin n \le 1$$

$$-1 \le (-1)^n \sin(n) \le 1$$

$$-\frac{1}{\sqrt{n}} \le (-1)^n \frac{\sin(n)}{\sqrt{n}} \le \frac{1}{\sqrt{n}}$$

$$\lim_{n\to+\infty} -\frac{1}{\sqrt{n}} = 0 = \lim_{n\to+\infty} \frac{1}{\sqrt{n}} \Rightarrow \lim_{n\to+\infty} (-1)^n \cdot \frac{\sin n}{\sqrt{n}}$$

Teorem: $\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = r \Rightarrow \lim_{n\to+\infty} \sqrt[n]{a_n} = r$ 'dir.

9) $\left\{ \sqrt[n]{\frac{n!}{2^n}} \right\}$ yakınsak mıdır?

$$\lim_{n\to+\infty} \frac{\frac{(n+1)!}{2^{n+1}}}{\frac{n!}{2^n}} = \lim_{n\to+\infty} \frac{\cancel{(n+1)!}^{\,n+1}}{\cancel{n!}} \frac{\cancel{2^n}}{2^{\cancel{n}+1}} = \lim_{n\to+\infty} \frac{n+1}{2}$$

$= +\infty$ Teoremden $\lim_{n\to+\infty} \sqrt[n]{\frac{n!}{2^n}} = +\infty$ ıraksaktır.

10) $\left\{ \frac{n!}{n^n} \right\}$ yakınsak mıdır.

$\forall n \in \mathbb{N}$ için $0 < \frac{n!}{n^n} = \frac{1.2.3 \dots n}{n.n.n \dots n} < \frac{1}{n}$

$$\lim_{n\to+\infty} 0 = 0 = \lim_{n\to+\infty} \frac{1}{n} \Rightarrow \lim_{n\to+\infty} \frac{n!}{n^n} = 0$$

11) $\left\{ \sqrt{7}, \sqrt{7.\sqrt{7}}, \sqrt{7.\sqrt{7.\sqrt{7}}}, \dots, \sqrt{7.a_{n-1}}, \dots \right\}$

$a_1 = \sqrt{7}$, $a_n = \sqrt{7a_{n-1}}$

$$\lim_{n\to+\infty} a_n = \sqrt{7\sqrt{7\sqrt{7\sqrt{7 \dots}}}} = x$$

$$7 \underbrace{\sqrt{7\sqrt{7\sqrt{7 \dots}}}}_{x} = x^2$$

$7x = x^2 \Rightarrow x^2 - 7x = 0 \Rightarrow x(x-7) = 0$ $\quad \cancel{x=0}$ $\boxed{x=7}$ ✓ yakınsak

Teorem:

i) Her $n \geq N$ için $a_n \leq a_{n+1}$ ve $\{a_n\}$ üstten sınırlı ise yakınsaktır.

ii) " " " $a_n \geq a_{n+1}$ ve $\{a_n\}$ alttan " " " .

12) $\left\{ \frac{2.4.6 \ldots .2n}{3.5.7 \ldots (2n+1)} \right\}$ yakınsak mıdır.

$\forall n \in \mathbb{N}$ için $0 < \frac{2.4.6 \ldots 2n}{3.5.7 \ldots 2n+1}$ olup

dizi alttan 0 ile sınırlıdır.

$\forall n \in \mathbb{N}$ için $\dfrac{\dfrac{\cancel{2}.\cancel{4}.\cancel{6} \ldots \cancel{2n}(2n+2)}{\cancel{3}.\cancel{5}.\cancel{7} \ldots (\cancel{2n+1})(2n+3)}}{\dfrac{\cancel{2}.\cancel{4}.\cancel{6} \ldots \cancel{2n}}{\cancel{3}.\cancel{5}.\cancel{7} \ldots (\cancel{2n+1})}} = \frac{2n+2}{2n+3} < 1$

$\frac{a_{n+1}}{a_n} < 1 \Rightarrow a_{n+1} < a_n \Rightarrow a_n > a_{n+1}$

$a_n \geq a_{n+1}$

Teorem ii'den $\left\{ \frac{2.4.6 \ldots 2n}{3.5.7 \ldots (2n+1)} \right\}$ yakınsak.

Alıştırmalar:

Aşağıdaki dizilerin yakınsaklık-ıraksaklık durumlarını inceleyiniz.

$\left\{ \frac{\sin(\frac{2}{n})}{\sin(\frac{n+1}{n^2})} \right\}$, $\left\{ \frac{3^{n+1} - 2^n}{4^n + 3^n} \right\}$, $\left\{ (-1)^n \frac{n+1}{2n+3} \right\}$,

$\left\{ \sqrt[3n]{n!} \right\}$, $\left\{ \sqrt[n]{\frac{n+1}{2}} \right\}$, $\{\cos(n)\}$, $\{\frac{1}{n}\cos(n)\}$