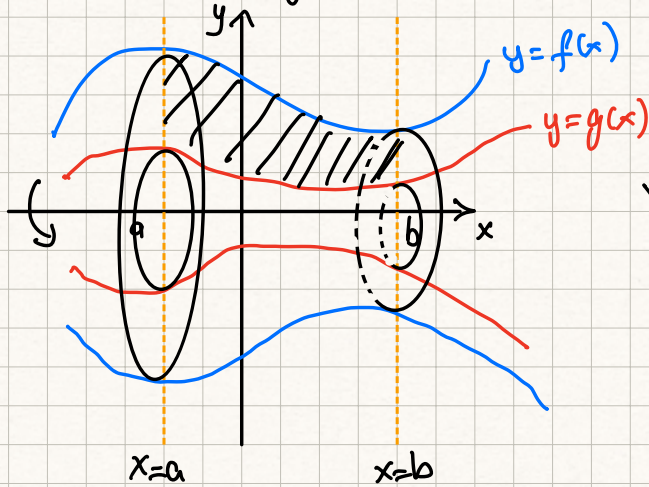


2) Hacim Hesabı:

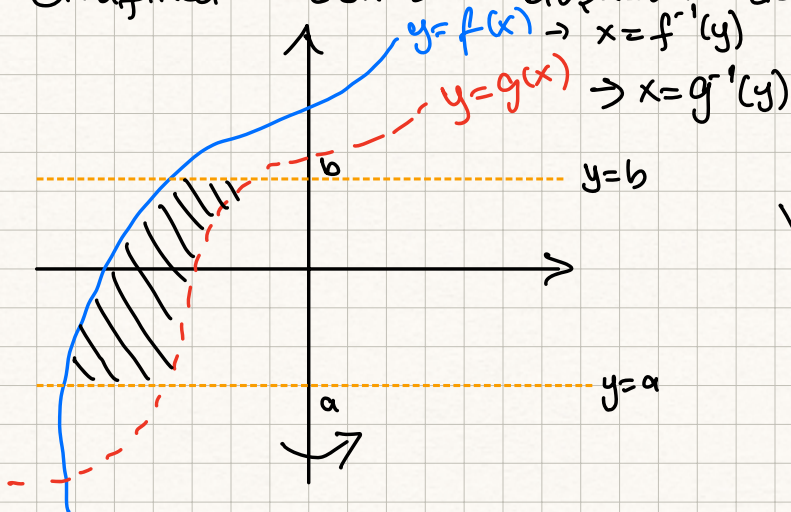
Disk Yöntemi:

$y=f(x)$, $y=g(x)$, $x=a$, $x=b$ ile sınırlı bölge x eksenini etrafında döndüğünde oluşacak dönel cismin hacmi



$$V = \pi \int_a^b (f^2(x) - g^2(x)) dx \text{ ile hesaplanır.}$$

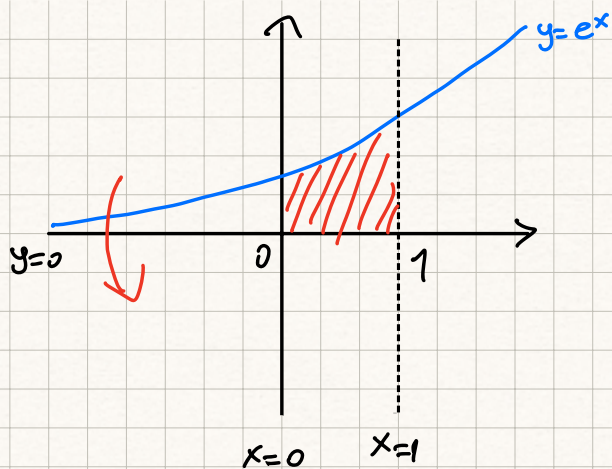
$y=f(x)$, $y=g(x)$, $y=a$, $y=b$ ile sınırlı bölge y eksenini etrafında dönerse oluşacak dönel cismin hacmi



$$V = \pi \int_a^b ((f^{-1}(y))^2 - (g^{-1}(y))^2) dy$$

Örnekler:

1) $y=e^x$, $x=0$, $x=1$, $y=0$, x eksenini

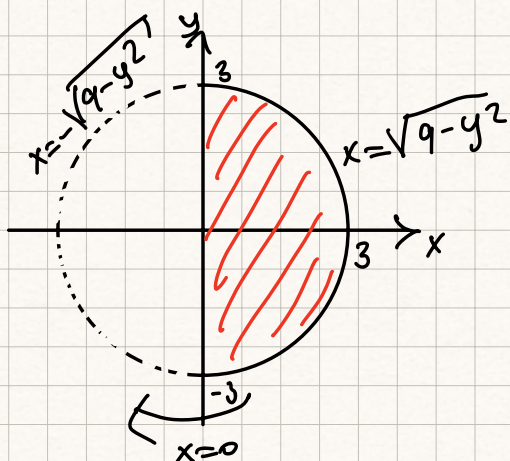


$$V = \pi \int_0^1 (e^x)^2 - 0^2 dx = \pi \int_0^1 e^{2x} dx$$

$$= \pi \left(\frac{e^{2x}}{2} \right) \Big|_0^1$$

$$= \pi \left(\frac{e^2}{2} - \frac{1}{2} \right) \text{ br}^3$$

2) $x = \sqrt{9-y^2}$, $x=0$, y eksenini



$$V = \pi \int_{-3}^3 (\sqrt{9-y^2})^2 - 0^2 dy$$

$$= \pi \int_{-3}^3 9-y^2 dy$$

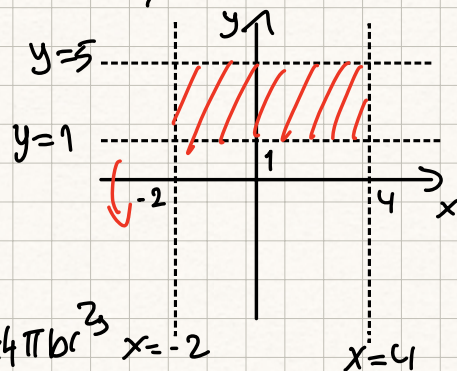
$$= \pi \left(9y - \frac{y^3}{3} \right) \Big|_{-3}^3$$

$$= \pi \left((27-9) - (-27+9) \right) = \pi 36 \text{ br}^3$$

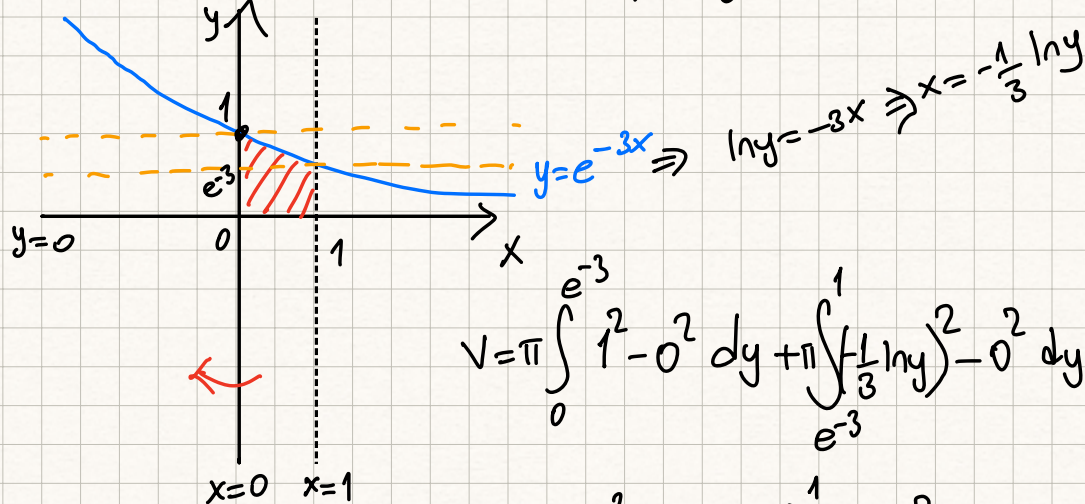
3) $y=1$, $y=5$, $x=-2$, $x=4$, x eksenini

$$V = \pi \int_{-2}^4 5^2 - 1^2 dx = 24\pi \int_{-2}^4 dx$$

$$= 24\pi \left(x \right) \Big|_{-2}^4 = 24\pi (4 - (-2)) = 144\pi \text{ br}^3$$



4) $y = e^{-3x}$, $x=0$, $x=1$, $y=0$, y eksenini



$$V = \pi \int_0^{e^{-3}} 1^2 - 0^2 dy + \pi \int_{e^{-3}}^1 \left(-\frac{1}{3} \ln y\right)^2 - 0^2 dy$$

$$= \pi e^{-3} + \frac{\pi}{9} \int_{e^{-3}}^1 (\ln y)^2 dy$$

$$= \pi e^{-3} + \frac{\pi}{9} \left(y \ln^2 y - 2(y \ln y - y) \right) \Big|_{e^{-3}}^1$$

$$= \pi e^{-3} + \frac{\pi}{9} \left(2 - \left(e^{-3} 9 - 2(e^{-3}(-3) - e^{-3}) \right) \right)$$