Çözümler:

$$* \sum_{n=1}^{+\infty} \frac{5^n}{n!}(x+2)^n$$

$$\lim_{n\to+\infty}\left|\frac{\frac{5^{\cancel{n+1}}}{(n+1)!}(x+2)^{\cancel{n+1}}}{\frac{\cancel{5^n}}{n!}\cancel{(x+2)^n}}\right| = \lim_{n\to+\infty}\left|5\,\frac{\cancel{n!}}{\underbrace{\cancel{(n+1)!}}_{n+1}}(x+2)\right|$$

$$=\lim_{n\to+\infty}\frac{5}{n+1}|x+2| = 0\cdot|x+2| = 0 < 1$$

Y.M $= -2$

Y.Y $= +\infty$

Y.A $= (-\infty,+\infty) = \mathbb{R}$

$$* \sum_{n=2}^{+\infty} \frac{(-1)^n(x-1)^n}{n\ln n}$$

$$\lim_{n\to+\infty}\left|\frac{\frac{(-1)^{\cancel{n+1}}(x-1)^{\cancel{n+1}}}{(n+1)\ln(n+1)}}{\frac{\cancel{(-1)^n}\,\cancel{(x-1)^n}}{n\cdot\ln n}}\right| = \lim_{n\to+\infty}\left|\frac{n}{n+1}\,\frac{\ln n}{\ln(n+1)}(x-1)\right|$$

$$=\lim_{n\to+\infty}\underbrace{\frac{n}{n+1}}_{1}\,\underbrace{\frac{\ln n}{\ln(n+1)}}_{1}\,\underbrace{|x-1|}_{|x-1|} = |x-1| < 1$$

$$\Rightarrow -1 < x-1 < 1$$

$$\Rightarrow \boxed{0 < x < 2}$$

$$x=0 \Rightarrow \sum_{n=2}^{+\infty}\frac{(-1)^n}{n\cdot\ln n}(0-1)^n = \sum_{n=2}^{+\infty}\frac{1}{n\cdot\ln n} = \sum_{n=1}^{+\infty}\frac{1}{(n+1)\ln(n+1)}$$

$\left\{\frac{1}{(n+1)\cdot\ln(n+1)}\right\}$ azalan bir dizidir.

$f:[1,+\infty)\to[0,+\infty)$, $f(x)=\dfrac{1}{(x+1)\ln(x+1)}$ sürekli ve azalandır. O halde

$$\int_1^{+\infty}\frac{1}{(x+1)\ln(x+1)}\,dx \qquad \sum_{n=1}^{+\infty}\frac{1}{(n+1)\ln(n+1)} \text{ aynı karakterli}$$

$$\int_1^{+\infty}\frac{1}{(x+1)\ln(x+1)}\,dx=\lim_{t\to+\infty}\int_1^{t}\frac{1}{(x+1)\ln(x+1)}\,dx$$

$$=\lim_{t\to+\infty}\ln(\ln(x+1))\Big|_1^t=\lim_{t\to+\infty}\underbrace{\ln(\underbrace{\ln(\underbrace{t+1}_{+\infty})}_{+\infty})}_{+\infty}-\ln(\ln 2)=+\infty$$

ıraksak olduğundan integral testinden $\displaystyle\sum_{n=1}^{+\infty}\frac{1}{(n+1)\ln(n+1)}$ ıraksaktır.

$$\int\frac{1}{(x+1)\ln(x+1)}\,dx=\int\frac{1}{u}\,du=\ln|u|+c=\ln|\ln(x+1)|+c$$

$$=\ln(\ln(x+1))+c \qquad \boxed{\begin{array}{l}\ln(x+1)=u\\ \frac{1}{x+1}dx=du\end{array}}$$

$x=2 \Rightarrow \displaystyle\sum_{n=2}^{+\infty}\frac{(-1)^n(2-1)^n}{n\ln(n)}=\sum_{n=2}^{+\infty}(-1)^n\frac{1}{n\ln n}$ serisi için

$\forall n\in\mathbb{N}\setminus\{2\}$ $\quad 0<\dfrac{1}{(n+1)\ln(n+1)}\leq\dfrac{1}{n\ln n}$

ve $\displaystyle\lim_{n\to+\infty}\frac{1}{n\ln(n)}=0 \Rightarrow$ Leibnitz Testinden yakınsaktır.

Y.M $=1$

Y.Y $=1$

Y.A $=(0,2]$

* $\displaystyle\sum_{n=1}^{+\infty} \frac{(4x-5)^{\frac{n}{3}}}{3^n}$ $\qquad \displaystyle\sum_{n=1}^{+\infty} \frac{3^n}{3^n} = \sum_{n=1}^{+\infty} 1 \qquad \sum_{n=1}^{+\infty} (-1)^n$

$$\lim_{n\to+\infty} \left| \frac{\dfrac{(4x-5)^{\frac{\cancel{n}+1}{3}}}{3^{\cancel{n}+1}}}{\dfrac{\cancel{(4x-5)^{\frac{n}{3}}}}{\cancel{3^n}}} \right| = \lim_{n\to+\infty} \left| \frac{(4x-5)^{1/3}}{3} \right| = \left| \frac{(4x-5)^{1/3}}{3} \right| < 1$$

$$\Rightarrow \quad |4x-5|^{1/3} < 3$$

$$|4x-5| < 27$$

$$-27 < 4x-5 < 27$$

$$-22 < 4x < 32$$

$$-5{,}5 < x < 8$$

$x = 5{,}5 \Rightarrow \displaystyle\sum_{n=1}^{+\infty} \frac{(4.(-5{,}5)-5)^{\frac{n}{3}}}{3^n} = \sum_{n=1}^{+\infty} \frac{(-27)^{\frac{n}{3}}}{3^n} = \sum_{n=1}^{+\infty} \frac{(-3)^n}{3^n} = \sum_{n=1}^{+\infty} (-1)^n$

serisi için $\displaystyle\lim_{n\to+\infty} (-1)^n$ = limit yok $\neq 0 \Rightarrow$ G.T.T. ıraksaktır.

$x = 8 \Rightarrow \displaystyle\sum_{n=1}^{+\infty} \frac{(4.8-5)^{\frac{n}{3}}}{3^n} = \sum_{n=1}^{+\infty} \frac{(27)^{\frac{n}{3}}}{3^n} = \sum_{n=1}^{+\infty} \frac{3^n}{3^n} = \sum_{n=1}^{+\infty} 1$

serisi için $\displaystyle\lim_{n\to+\infty} 1 = 1 \neq 0 \Rightarrow$ G.T.T. ıraksaktır.

$Y.M = \frac{5}{4}$

$Y.Y = \frac{27}{4}$

$Y.A = (-5{,}5,\ 8)$

Taylor (Maclaurin) Serileri:

$f(x)$ fonksiyonunun bir $x_0 \in \mathbb{R}$'de her mertebeden türevi mevcut olsun.

$$\sum_{n=0}^{+\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n = f(x_0) + f^{(1)}(x_0)(x-x_0) + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \cdots$$

Kuvvet serisine $f(x)$ fonksiyonunun x_0 noktasındaki Taylor seri açılımı denir.

$x_0 = 0$ ise $\displaystyle\sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n$

kuvvet serisine $f(x)$ fonksiyonunun Maclaurin serisi denir.

Örnekler:

1) $f(x) = \sqrt[3]{\dfrac{7}{e^{\frac{4x-1}{2}}}}$, $x_0 = 3$'deki Taylor serisi?

$$f(x) = \sqrt[3]{7} \cdot \frac{1}{e^{\frac{4x-1}{6}}} = \sqrt[3]{7} \cdot e^{-\frac{2}{3}x + \frac{1}{3}}$$

$$f^{(1)}(x) = \sqrt[3]{7} \cdot \left(-\frac{2}{3}\right) \cdot e^{-\frac{2}{3}x + \frac{1}{3}}$$

$$f^{(2)}(x) = \sqrt[3]{7} \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) e^{-\frac{2}{3}x + \frac{1}{3}}$$

$$f^{(3)}(x) = \sqrt[3]{7} \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) e^{-\frac{2}{3}x + \frac{1}{3}}$$

$$\vdots$$

$$f^{(n)}(x) = \sqrt[3]{7} \cdot \left(-\frac{2}{3}\right)^n \cdot e^{-\frac{2}{3}x + \frac{1}{3}} \quad , \; n = 0, 1, 2, 3, \ldots, n, \ldots$$

$$f^{(n)}(3) = \sqrt[3]{7} \left(-\frac{2}{3}\right)^n e^{-\frac{5}{3}}$$

$$\sum_{n=0}^{+\infty} \frac{f^{(n)}(3)}{n!}(x-3)^n = \sum_{n=0}^{+\infty} \frac{\sqrt[3]{7}\left(-\frac{2}{3}\right)^n e^{-\frac{5}{3}}}{n!}(x-3)^n$$