Kuvvet Serileri:

$\{a_n\}$ bir dizi, $a \in \mathbb{R}$ ve $x \in \mathbb{R}$ bir değişken olsun.

$$\sum_{n=0}^{+\infty} a_n (x-a)^n = a_0 + a_1(x-a) + a_2(x-a)^2 + \dots + a_n(x-a)^n + \dots$$

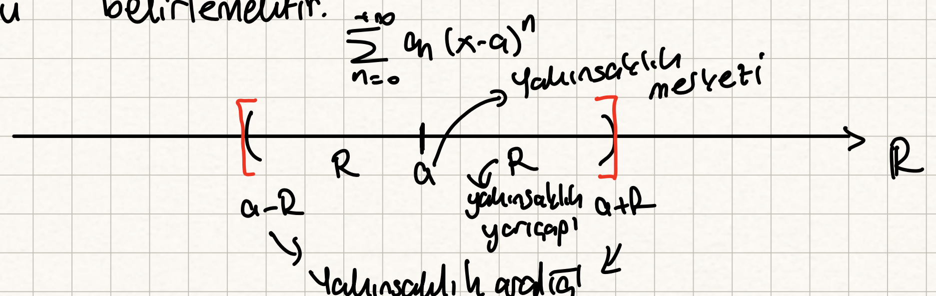
ifadesine bir kuvvet serisi denir.

$x=1 \Rightarrow \sum_{n=0}^{+\infty} a_n(1-a)^n$ bir reel sayı serisidir,

$x=-3 \Rightarrow \sum_{n=0}^{+\infty} a_n(-3-a)^n$ " " " " .

$x=a \Rightarrow \sum_{n=0}^{+\infty} a_n(a-a)^n = \sum_{n=0}^{+\infty} 0$ " " " " .

Kuvvet serilerindeki asıl problem, x yerine hangi reel sayılar yazıldığında elde edilen reel sayı serilerinin yakınsak olduğunu belirlemektir.



$Y.M = a$

$Y.Y = R$

$Y.A = (a-R, a+R)$
$\quad\quad\;\; [\quad\quad\quad\;]$

Örnekler:

1) $\sum_{n=1}^{+\infty} (-1)^n \frac{\ln n}{n} (x-2)^n$ için

Y.M = ?
Y.Y = ?
Y.A = ?

Y.M = 2'dir.

$$\lim_{n\to+\infty} \left| \frac{\cancel{(-1)^{n+1}} \frac{\ln(n+1)}{n+1} \cancel{(x-2)^{n+1}}}{\cancel{(-1)^n} \frac{\ln(n)}{n} \cancel{(x-2)^n}} \right| = \lim_{n\to+\infty} \left| (-1) \frac{\ln(n+1)}{\ln n} \frac{n}{n+1} (x-2) \right|$$

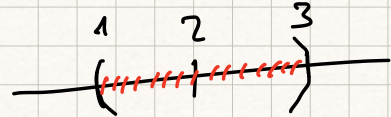
$$= \lim_{n\to+\infty} \frac{\ln(n+1)}{\ln(n)} \frac{n}{n+1} |x-2| = |x-2| \lim_{n\to+\infty} \underbrace{\frac{\ln(n+1)}{\ln n}}_{1} \underbrace{\frac{n}{n+1}}_{1}$$

$= |x-2| < 1$ için seri yakınsaktır.

$\Rightarrow -1 < x-2 < 1$ " " "

$\Rightarrow 2-1 < x < 2+1$

$\Rightarrow 1 < x < 3$ " " "



$\boxed{Y.Y = 1}$

$x=1$ için $\sum_{n=1}^{+\infty} (-1)^n \frac{\ln n}{n} (1-2)^n = \sum_{n=1}^{+\infty} \frac{\ln n}{n}$

$\sum_{n=1}^{+\infty} \frac{1}{n}$ p testinden ıraksak

$\forall n \in \mathbb{N} \setminus \{1\}$ $\frac{1}{n} < \frac{\ln n}{n}$ old. Karşılaştırma T

$\sum_{n=1}^{+\infty} \frac{1}{n}$ ıraksak $\Rightarrow \sum_{n=1}^{+\infty} \frac{\ln n}{n}$ ıraksaktır.

$x=3 \Rightarrow \sum_{n=1}^{+\infty} (-1)^n \frac{\ln n}{n} \underbrace{(3-2)^n}_{1} = \sum_{n=1}^{+\infty} (-1)^n \frac{\ln n}{n}$ Alterne seri

$\forall n \geq 3$ için $0 < \frac{\ln(n+1)}{n+1} \leq \frac{\ln(n)}{n}$ ve $\lim\limits_{n\to+\infty} \frac{\ln n}{n} \overset{\frac{\infty}{\infty}}{=} 0$

Leibnitz T. $\sum_{n=1}^{+\infty} (-1)^n \frac{\ln n}{n}$ yakınsaktır.

$Y.M = 2$

$Y.Y = 1$

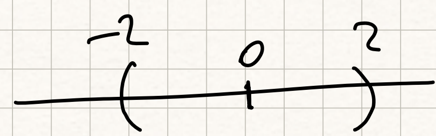
$Y.A = (1, 3]$ dir.

2) $\sum_{n=1}^{+\infty} \frac{n}{2^n(n^2+1)} x^n$ ---

$$\lim_{n\to+\infty} \left| \frac{\frac{n+1}{2^{\cancel{n+1}}((n+1)^2+1)} x^{\cancel{n+1}}}{\frac{n}{\cancel{2^n}(n^2+1)} \cancel{x^n}} \right| = \lim_{n\to+\infty} \left| \frac{n+1}{n} \frac{n^2+1}{(n+1)^2+1} \frac{1}{2} x \right|$$

$$= \lim_{n\to+\infty} \frac{n+1}{n} \frac{n^2+1}{(n+1)^2+1} \frac{1}{2} |x| = 1.1.\frac{1}{2}.|x| = \frac{|x|}{2} < 1$$

$\Rightarrow |x| < 2 \Rightarrow -2 < x < 2$



$x=-2 \Rightarrow \sum_{n=1}^{+\infty} \frac{n}{2^n(n^2+1)} (-2)^n = \sum_{n=1}^{+\infty} (-1)^n \frac{n}{n^2+1}$ Alterne seri

$\forall n \in \mathbb{N}$ $0 < \frac{n+1}{(n+1)^2+1} \leq \frac{n}{n^2+1}$, $\lim\limits_{n\to+\infty} \frac{n}{n^2+1} = 0$

Leibnitz T seri yakınsaktır.

$x=2 \Rightarrow \sum_{n=1}^{+\infty} \frac{n}{2^n(n^2+1)} 2^n = \sum_{n=1}^{+\infty} \frac{n}{n^2+1}$

$$\frac{n}{n^2+1} \geqslant \frac{n}{n^2+n^2} = \frac{n}{2n^2} = \frac{1}{2} \cdot \frac{1}{n}$$

$\sum_{n=1}^{+\infty} \frac{1}{2} \cdot \frac{1}{n} = \frac{1}{2} \sum_{n=1}^{+\infty} \frac{1}{n}$ p testi ıraksak

Karşılaştırma T. $\sum_{n=1}^{+\infty} \frac{n}{n^2+1}$ ıraksaktır.

$Y.M = 0$

$Y.Y = 2$

$Y.A = [-2, 2)$

3) $\sum_{n=1}^{+\infty} \frac{(x+1)^n}{\sqrt{n}}$ - - - -

$$\lim_{n\to+\infty} \left| \frac{\frac{(x+1)^{\cancel{n+1}}}{\sqrt{n+1}}}{\frac{\cancel{(x+1)^n}}{\sqrt{n}}} \right| = \lim_{n\to+\infty} \left| \frac{\sqrt{n}}{\sqrt{n+1}} (x+1) \right|$$

$$= \lim_{n\to+\infty} \frac{\sqrt{n}}{\sqrt{n+1}} |x+1| = 1 \cdot |x+1| < 1$$

$\Rightarrow -1 < x+1 < 1 \quad \Rightarrow -2 < x < 0$

$x=-2$ için $\sum_{n=1}^{+\infty} \frac{(-2+1)^n}{\sqrt{n}} = \sum_{n=1}^{+\infty} (-1)^n \cdot \frac{1}{\sqrt{n}}$ Alterne seri

$\forall n \in \mathbb{N} \quad 0 < \frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$, $\lim_{n \to +\infty} \frac{1}{\sqrt{n}} = 0$

Leibnitz T. seri yakınsaktır.

$x=0$ için $\sum_{n=1}^{+\infty} \frac{(0+1)^n}{\sqrt{n}} = \sum_{n=1}^{+\infty} \frac{1}{\sqrt{n}}$ p-Testi ıraksak.

$Y.M = -1$

$Y.Y = 1$

$Y.A = [-2, 0)$