5) $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ yakınsak mıdır?

$$\left\{\left(1+\frac{1}{n}\right)^n\right\} = \left\{2, \frac{9}{4}, \frac{64}{27}, \dots, \left(1+\frac{1}{n}\right)^n, \dots\right\}$$

$$\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n \overset{1^\infty}{=} e^1 = e \text{ yakınsaktır.} \qquad \boxed{1^\infty, 0^0, \infty^0}$$

$$\lim_{n\to+\infty} n \cdot \ln\left(1+\frac{1}{n}\right) \overset{0\cdot\infty}{=} \lim_{n\to+\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} \overset{\frac{0}{0}}{=}$$

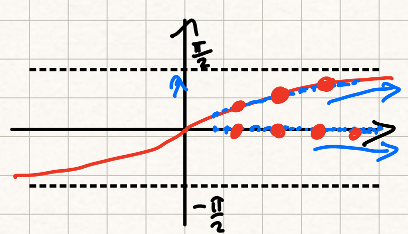
$$= \lim_{n\to+\infty} \frac{\cancel{-\frac{1}{n^2}} \cdot \frac{1}{1+\frac{1}{n}}}{\cancel{-\frac{1}{n^2}}} = \frac{1}{1+0} = 1$$

6) $\left\{n \cdot \sin\left(\frac{6}{n}\right)\right\}$ yakınsak mıdır.

$$\lim_{n\to+\infty} n \cdot \sin\left(\frac{6}{n}\right) \overset{0\cdot\infty}{=} \lim_{n\to+\infty} \frac{\sin\left(\frac{6}{n}\right)}{\frac{1}{n}} \overset{\frac{0}{0}}{=} \lim_{n\to+\infty} \frac{\cancel{\frac{-6}{n^2}} \cdot \cos\left(\frac{6}{n}\right)}{\cancel{-\frac{1}{n^2}}}$$

$$= \lim_{n\to+\infty} 6 \cdot \cos\left(\frac{6}{n}\right) = 6 \cdot \cos 0 = 6 \text{ yakınsaktır.}$$

7) $$\lim_{n\to+\infty} \frac{\pi}{4} \arctan n = \frac{\pi}{4} \cdot \frac{\pi}{2} = \frac{\pi^2}{8} \text{ yakınsaktır}$$



Teorem: $\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = r \Rightarrow \lim_{n\to+\infty} \sqrt[n]{a_n} = r$ 'dir.

8) $\left\{ \sqrt[n]{\frac{1+n^2}{3n+2}} \right\} = \left\{ \left(\frac{1+n^2}{3n+2}\right)^{\frac{1}{n}} \right\}$

$$\lim_{n\to+\infty} \frac{\frac{1+(n+1)^2}{3(n+1)+2}}{\frac{1+n^2}{3n+2}} = \lim_{n\to+\infty} \frac{1+(n+1)^2}{1+n^2} \cdot \frac{3n+2}{3(n+1)+2} = 1.1 = 1$$

Teoremden $\lim_{n\to+\infty} \sqrt[n]{\frac{1+n^2}{3n+2}} = 1$

9) $\left\{ \sqrt[2n]{\frac{n!}{n^n}} \right\}$ yakınsak mıdır? $\left(\sqrt[2n]{\frac{n!}{n^n}} = \left(\frac{n!}{n^n}\right)^{\frac{1}{2n}} = \sqrt[n]{\left(\frac{n!}{n^n}\right)^{1/2}} = \sqrt[n]{\sqrt{\frac{n!}{n^n}}} \right)$

$$\lim_{n\to+\infty} \frac{\sqrt{\frac{(n+1)!}{(n+1)^{n+1}}}}{\sqrt{\frac{n!}{n^n}}} = \lim_{n\to+\infty} \sqrt{\frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}}} = \lim_{n\to+\infty} \sqrt{\frac{\cancel{(n+1)!}^{\,n+1}}{\cancel{n!}} \frac{n^n}{(n+1)^{n+\cancel{1}}}}$$

$$= \lim_{n\to+\infty} \sqrt{\frac{n^n}{(n+1)^n}} = \lim_{n\to+\infty} \sqrt{\left(\frac{n}{n+1}\right)^n} = \lim_{n\to+\infty} \left(\frac{n}{n+1}\right)^{\frac{n}{2}} \overset{1^\infty}{=} e^{-1/2}$$

$$\lim_{n\to+\infty} \frac{n}{2} \ln\left(\frac{n}{n+1}\right) \overset{\infty.0}{=} \lim_{n\to+\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{2}{n}} \quad \frac{0}{0}$$

$$= \lim_{n\to+\infty} \frac{\frac{\cancel{1.(n+1)-1.n}}{(n+1)^{\cancel{2}}} \cdot \frac{1}{\frac{\cancel{n}}{\cancel{n+1}}}}{-\frac{2}{n^{\cancel{2}}}} = \lim_{n\to+\infty} \frac{-n}{2(n+1)} = -\frac{1}{2}$$

Teoremden

$$\lim_{n\to+\infty} \sqrt[n]{\sqrt{\frac{n!}{n^n}}} = e^{-1/2} \Rightarrow \lim_{n\to+\infty} \sqrt[2n]{\frac{n!}{n^n}} = \frac{1}{\sqrt{e}}$$

Teorem: Bir $N \in \mathbb{N}$ mevcut olsun, öyleki $n \geq N$ olan her $n \in \mathbb{N}$ için $a_n \leq b_n \leq c_n$ ve $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} c_n = L \Rightarrow \lim_{n\to+\infty} b_n = L$ dir.

10) $\left\{ \frac{n!}{n^n} \right\}$

$$\frac{n!}{n^n} = \frac{1.2.3.\cdots.n}{n.n.n.\cdots.n} = \frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{n}{n} \leq 1$$

Her $n \in \mathbb{N}$ için

$$0 < \frac{2}{n}\cdot\frac{3}{n}\cdot\frac{4}{n}\cdots\frac{n}{n} \leq 1$$

olduğundan $0 < \frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{n}{n} \leq \frac{1}{n}$ doğrudur.

$\lim_{n\to+\infty} 0 = 0 = \lim_{n\to+\infty} \frac{1}{n}$ Teoremden $\lim_{n\to+\infty} \frac{n!}{n^n} = 0$

11) $\left\{ \frac{\cos^2 n}{4^n} \right\}$ yakınsak mıdır?

Her $n \in \mathbb{N}$ için $-1 \leq \cos n \leq 1 \Rightarrow 0 \leq \cos^2 n \leq 1$

$$\Rightarrow 0 \leq \frac{\cos^2 n}{4^n} \leq \frac{1}{4^n}$$

$\lim_{n\to+\infty} 0 = 0 = \lim_{n\to+\infty} \frac{1}{4^n}$ Teoremden $\lim_{n\to+\infty} \frac{\cos^2 n}{4^n} = 0$

12) $a_1=\sqrt{7}$, $a_n=\sqrt{7+a_{n-1}}$ $\Rightarrow$ $\lim\limits_{n\to+\infty} a_n = ?$

$$\{a_n\}=\{\sqrt{7}, \sqrt{7+\sqrt{7}}, \sqrt{7+\sqrt{7+\sqrt{7}}}, \dots\}$$

$$\lim_{n\to+\infty} a_n = \sqrt{7+\sqrt{7+\sqrt{7+\sqrt{\dots}}}} = x = \frac{1+\sqrt{29}}{2}$$

$$7+\sqrt{7+\sqrt{7+\sqrt{7+\sqrt{\dots}}}} = x^2$$

$$7+x=x^2$$

$$x^2-x-7=0$$

$$\Delta=(-1)^2-4\cdot1\cdot(-7)=29$$

$$\Rightarrow x=\frac{1\pm\sqrt{29}}{2} \Rightarrow x=\frac{1+\sqrt{29}}{2}$$

13) $\lim\limits_{n\to+\infty} \sqrt[n]{\dfrac{1.3.5\dots2n-1}{2\,4\,6\dots2n}} \stackrel{\text{Teo}}{=} 1$

$$\lim_{n\to+\infty} \frac{\dfrac{\cancel{1}.\cancel{3}.\cancel{5}\dots\cancel{(2n-1)}(2n+1)}{\cancel{2}\cdot\cancel{4}\cdot\cancel{6}\dots\cancel{2n}\cdot(2n+2)}}{\dfrac{\cancel{1}.\cancel{3}.\cancel{5}\dots\cancel{(2n-1)}}{\cancel{2}\cdot\cancel{4}\cdot\cancel{6}\dots\cancel{2n}}} = \lim_{n\to+\infty}\frac{2n+1}{2n+2}=1$$

Teorem:

i) Her $n\geq N$ için $a_n\leq a_{n+1}$ ve $\{a_n\}$ üstten sınırlı ise yakınsaktır.

ii) " " " $a_n\geq a_{n+1}$ ve $\{a_n\}$ alttan " " " .

14) $\lim\limits_{n\to+\infty} \underbrace{\dfrac{1.3.5\dots2n-1}{2.4.6\dots2n}}_{a_n}$?

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{\cancel{1}.\cancel{3}.\cancel{5}.\ldots\cancel{(2n-1)}.(2n+1)}{\cancel{2}.\cancel{4}.\cancel{6}.\ldots\cancel{2n}(2n+2)}}{\dfrac{\cancel{1}.\cancel{3}.\cancel{5}.\ldots\cancel{(2n-1)}}{\cancel{2}.\cancel{4}.\cancel{6}.\ldots\cancel{2n}}} = \frac{2n+1}{2n+2} \leq 1$$

olduğundan $a_n = \dfrac{1.3.5.\ldots(2n-1)}{2.4.6.\ldots(2n)}$ azalan bir dizidir.

Her $n \in \mathbb{N}$ için $a_n = \dfrac{1.3.5.\ldots(2n-1)}{2.4.6.\ldots2n} \geq 0$ dır.

Teorem gereği $\{a_n\}$ yakınsaktır.