

Çok Değişkenli Fonksiyonlar:

$(x_1, x_2, \dots, x_n)$ sıralı n 'li terimi $F(x_1, x_2, \dots, x_n)$ reel sayısına götüren F 'ye bir n değişkenli fonksiyon denir.

$$\underbrace{\mathbb{R} \times \mathbb{R} \times \dots \times \mathbb{R}}_{n \text{ tane}} = \mathbb{R}^n = \{ (x_1, x_2, \dots, x_n) \mid x_1, x_2, \dots, x_n \in \mathbb{R} \}$$

$F: \mathbb{R}^n \rightarrow \mathbb{R}$ n değişkenli fonksiyondur.

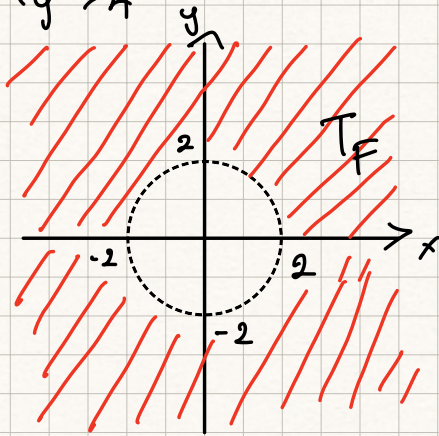
Tanım Kümesi:

İki değişkenli bir fonksiyonun tanım kümesi düzlemde bir bölgedir.

Örnek: $F(x, y) = \ln(x^2 + y^2 - 4)$ fonksiyonunun tanım kümesi?

$$x^2 + y^2 - 4 > 0 \text{ olmalı}$$

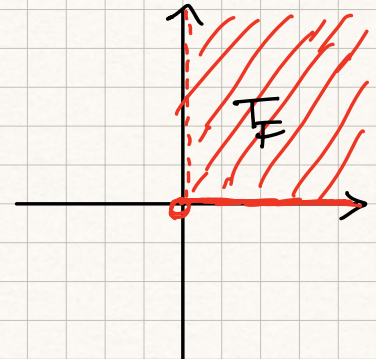
$$x^2 + y^2 > 4$$



Örnek: $F(x, y) = \sqrt{x} + \ln(y)$ $\Rightarrow T_F = ?$

$$x \geq 0 \text{ ve } y > 0$$

$$T_F = \{ (x, y) \in \mathbb{R}^2 \mid x \geq 0, y > 0 \}$$



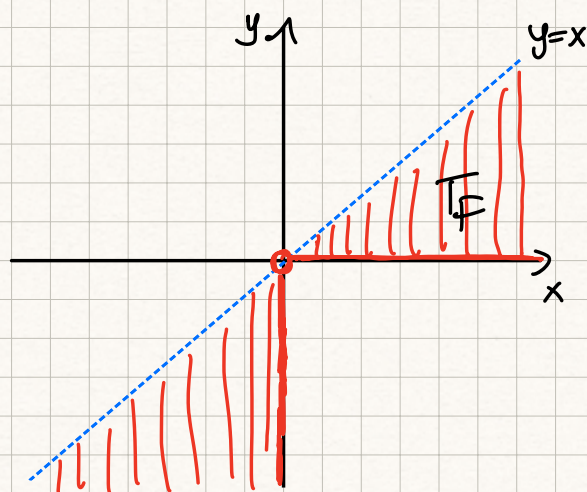
Örnek: $F(x,y) = \sqrt{xy} + \ln(x-y) \Rightarrow T_F = ?$

$xy \geq 0$ ve $x-y > 0$

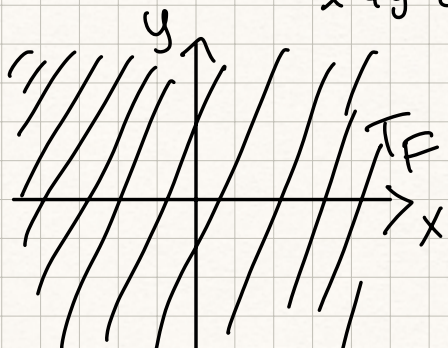
$T_F = \{(x,y) \in \mathbb{R}^2 \mid xy \geq 0 \text{ ve } x-y > 0\}$

$x \cdot y \geq 0 \Rightarrow x \geq 0 \text{ ve } y \geq 0$
veya $x \leq 0 \text{ ve } y \leq 0$

$x-y > 0 \Rightarrow x > y$
 $\Rightarrow \underline{y < x}$

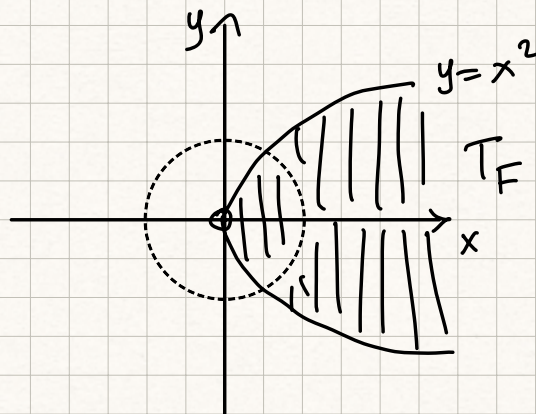
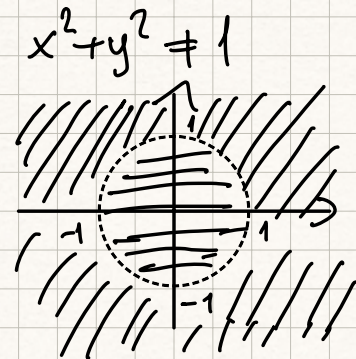
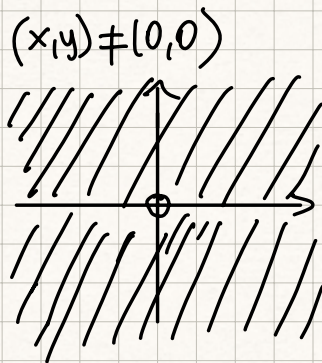
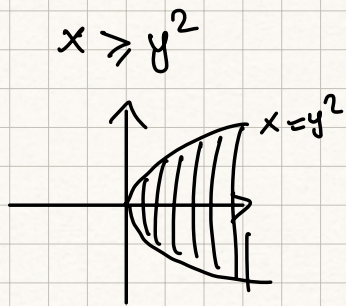


Örnek: $F(x,y) = \frac{e^{x+y^2} + \sin(xy)}{x^2+y^4 \cos 8x + 2y^4 + 1} \Rightarrow T_F = ?$



Örnek: $F(x,y) = \frac{\sqrt{x-y^2}}{\ln(x^2+y^2)} \Rightarrow T_F = ?$

$$x - y^2 \geq 0 \quad \vee \quad x^2 + y^2 > 0 \quad \vee \quad \ln(x^2 + y^2) \neq 0$$

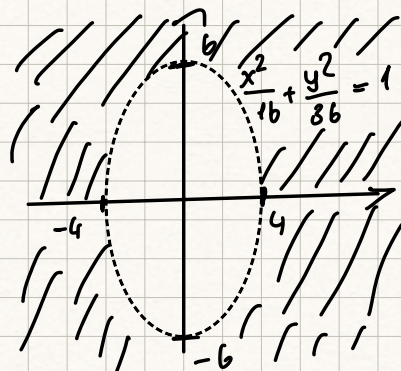


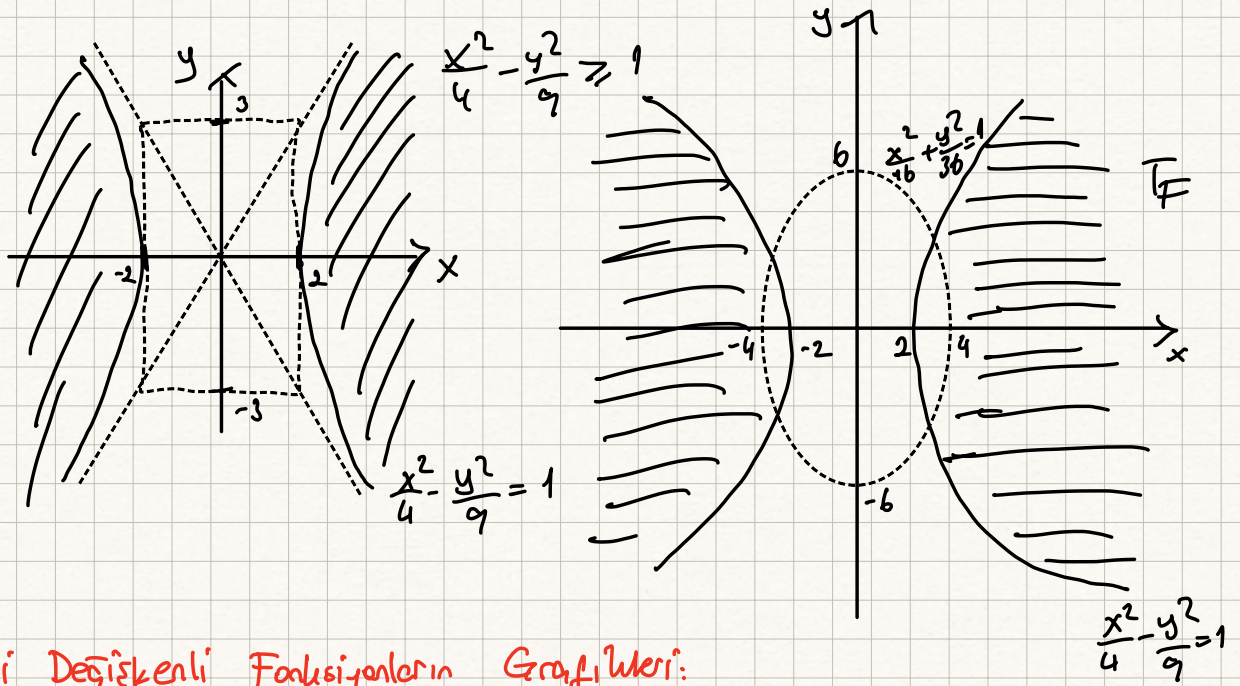
Örnek: $F(x, y) = \sqrt{\frac{x^2}{4} - \frac{y^2}{9} - 1} + \ln\left(\frac{x^2}{16} + \frac{y^2}{36} - 1\right) \Rightarrow T_F = ?$

$$\frac{x^2}{4} - \frac{y^2}{9} - 1 \geq 0 \quad \vee \quad \frac{x^2}{16} + \frac{y^2}{36} - 1 > 0$$

$$\frac{x^2}{4} - \frac{y^2}{9} \geq 1$$

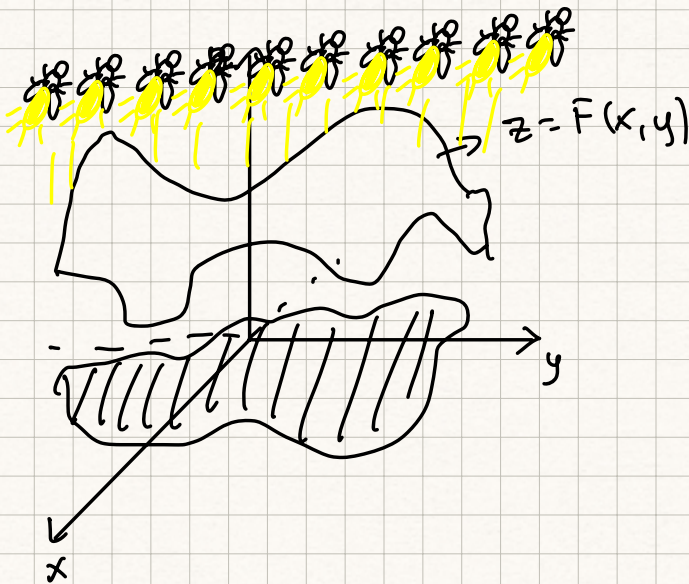
$$\frac{x^2}{16} + \frac{y^2}{36} > 1$$





İki Değişkenli Fonksiyonların Grafikleri:

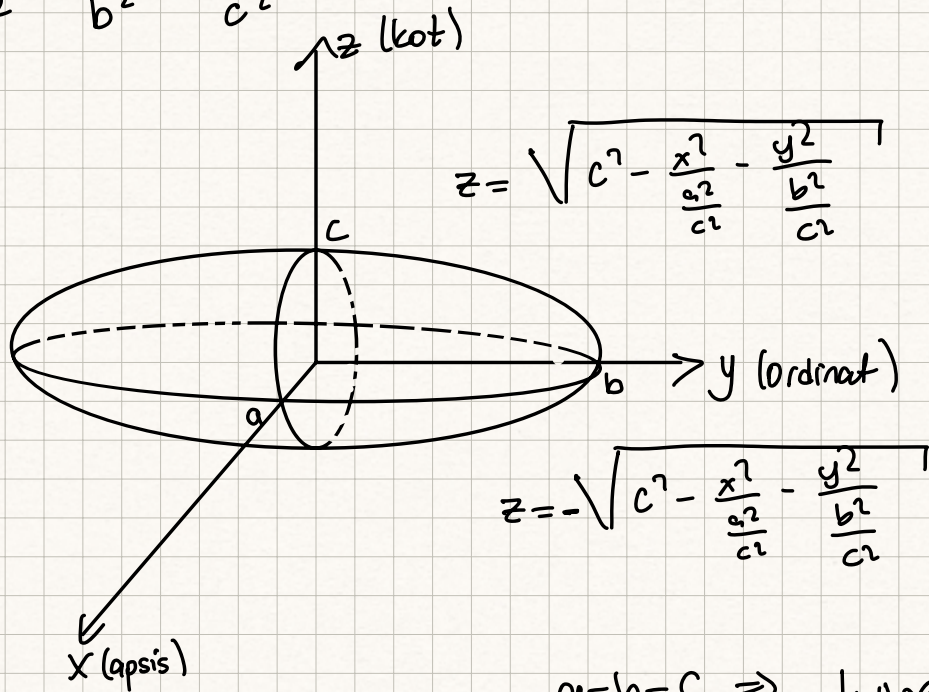
İki değişkenli bir fonksiyonun grafiği üç boyutlu uzayda bir yüzey belirler. $z = F(x, y)$ ile gösterilir.



Örnekler:

1) Ellipsoider:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad a, b, c \neq 0 \quad \Rightarrow \text{ellipsoid.}$$



$a=b=c \Rightarrow$ Kugel
edlir.