

Örnekler:

1) a) $\begin{cases} x+y=1 \\ 2x+2y=4 \end{cases}$

b) $\begin{cases} 3x-y=1 \\ 2x+y=4 \end{cases}$

c) $\begin{cases} 2x+4y=4 \\ x+2y=2 \end{cases}$ L.D.S
Çözünüz.

Çözüm:

a) $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \Rightarrow A \cdot X = B$

$$\underbrace{A^{-1} \cdot A}_{I} \cdot X = A^{-1} \cdot B$$

$$I \cdot X = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 4 \end{array} \right] \xrightarrow{-2S_1 \rightarrow S_2} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 2 \end{array} \right]$$

$$\xrightarrow{\frac{1}{2}S_2 \rightarrow S_2} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{-S_2 \rightarrow S_1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\text{rank}[A:B] = 2 > \text{rank}(A) = 1 \Rightarrow \text{Çözüm yok.}$$

b) $\left[\begin{array}{cc|c} 3 & -1 & 1 \\ 2 & 1 & 4 \end{array} \right] \xrightarrow{-S_2+S_1 \rightarrow S_1} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 2 & 1 & 4 \end{array} \right] \xrightarrow{-2S_1+S_2 \rightarrow S_2} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 5 & 10 \end{array} \right]$

$$\xrightarrow{\frac{1}{5}S_2 \rightarrow S_2} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 1 & 2 \end{array} \right] \xrightarrow{2S_2+S_1 \rightarrow S_1} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array} \right] \quad \boxed{\begin{matrix} x=1 \\ y=2 \end{matrix}} \quad X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\text{rank}[A:B] = 2 = \text{rank}(A) = n \Rightarrow \text{L.D.S Tek çözüm var.}$$

c) $\left[\begin{array}{cc|c} 2 & 4 & 4 \\ 1 & 2 & 2 \end{array} \right] \xrightarrow{\frac{1}{2}S_1 \rightarrow S_1} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 1 & 2 & 2 \end{array} \right] \xrightarrow{-S_1+S_2 \rightarrow S_2} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 0 \end{array} \right]$

$$\text{rank}[A:B] = 1 = \text{rank}(A) < 2 = n$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$2-1=1$ değişkene bağlı
sonsuz çözüm var.

$$\begin{cases} x+2y=2 \\ 0=0 \end{cases} \Rightarrow \begin{cases} x=2-2y \\ y=y \end{cases} \quad C_K = \left\{ \begin{bmatrix} 2-2y \\ y \end{bmatrix} \mid y \in \mathbb{R} \right\}$$

2) $\begin{cases} x-3y+2z=5 \\ 2x-y+z=-2 \\ 3x-z=4 \end{cases}$ L.D.S. çözümünüz.

$$\begin{bmatrix} 1 & -3 & 2 & | & 5 \\ 2 & -1 & 1 & | & -2 \\ 3 & 0 & -1 & | & 4 \end{bmatrix} \xrightarrow{\substack{-2S_1+S_2 \rightarrow S_2 \\ -3S_1+S_3 \rightarrow S_3}} \begin{bmatrix} 1 & -3 & 2 & | & 5 \\ 0 & 5 & -3 & | & -12 \\ 0 & 9 & -7 & | & -11 \end{bmatrix} \xrightarrow{2 \cdot S_2 \rightarrow S_2}$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & | & 5 \\ 0 & 10 & -6 & | & -24 \\ 0 & 9 & -7 & | & -11 \end{bmatrix} \xrightarrow{-S_2+S_3 \rightarrow S_3} \begin{bmatrix} 1 & -3 & 2 & | & 5 \\ 0 & 1 & 1 & | & -13 \\ 0 & 9 & -7 & | & -11 \end{bmatrix} \xrightarrow{-9S_2+S_3 \rightarrow S_3}$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & | & 5 \\ 0 & 1 & 1 & | & -13 \\ 0 & 0 & -16 & | & 106 \end{bmatrix} \xrightarrow{-\frac{1}{16}S_3 \rightarrow S_3} \begin{bmatrix} 1 & -3 & 2 & | & 5 \\ 0 & 1 & 1 & | & -13 \\ 0 & 0 & 1 & | & -\frac{53}{8} \end{bmatrix} \xrightarrow{\substack{-2S_3+S_1 \rightarrow S_1 \\ -S_3+S_2 \rightarrow S_2}}$$

$$\sim \begin{bmatrix} 1 & -3 & 0 & | & \frac{146}{8} \\ 0 & 1 & 0 & | & -\frac{51}{8} \\ 0 & 0 & 1 & | & -\frac{53}{8} \end{bmatrix} \xrightarrow{3S_2+S_1 \rightarrow S_1} \begin{bmatrix} 1 & 0 & 0 & | & -\frac{7}{8} \\ 0 & 1 & 0 & | & -\frac{51}{8} \\ 0 & 0 & 1 & | & -\frac{53}{8} \end{bmatrix} \quad \begin{aligned} &\frac{106}{8} + 5 \\ &-\frac{153}{8} + \frac{146}{8} \\ &= -\frac{7}{8} \end{aligned}$$

$$\text{rank}[A; B] = 3 = \text{rank}(A) = n$$

$\Rightarrow$ Tek çözüm var.

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{7}{8} \\ -\frac{51}{8} \\ -\frac{53}{8} \end{bmatrix}$$

$$x - 3y + 2z = 5 \Rightarrow -\frac{1}{4} + \frac{75}{4} - \frac{54}{4} = \frac{20}{4} = 5$$

$$2x - y + z = -2 \Rightarrow -\frac{2}{4} + \frac{25}{4} - \frac{27}{4} = \frac{4}{4} = 1 //$$

3)
$$\begin{cases} 3x - 5y + 2z + 4t = 2 \\ 7x - 4y + z + 3t = 5 \\ 5x + 7y - 4z - 6t = 3 \end{cases}$$
 L.D.S. Çözünüz.

$$\left[\begin{array}{cccc|c} 3 & -5 & 2 & 4 & 2 \\ 7 & -4 & 1 & 3 & 5 \\ 5 & 7 & -4 & -6 & 3 \end{array} \right] \xrightarrow{S_1 \leftrightarrow S_2} \left[\begin{array}{cccc|c} 7 & -4 & 1 & 3 & 5 \\ 3 & -5 & 2 & 4 & 2 \\ 5 & 7 & -4 & -6 & 3 \end{array} \right]$$

$$\xrightarrow{-2S_2 + S_1 \rightarrow S_1} \left[\begin{array}{cccc|c} 1 & 6 & -3 & -5 & 1 \\ 3 & -5 & 2 & 4 & 2 \\ 5 & 7 & -4 & -6 & 3 \end{array} \right] \xrightarrow{\begin{matrix} -3S_1 + S_2 \rightarrow S_2 \\ -5S_1 + S_3 \rightarrow S_3 \end{matrix}} \left[\begin{array}{cccc|c} 1 & 6 & -3 & -5 & 1 \\ 0 & -23 & 11 & 19 & -1 \\ 0 & -23 & 11 & 19 & -2 \end{array} \right]$$

$$\xrightarrow{-S_2 + S_3 \rightarrow S_3} \left[\begin{array}{cccc|c} 1 & 6 & -3 & -5 & 1 \\ 0 & -23 & 11 & 19 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{array} \right]$$

$\text{rank}[A;B] = 3 > \text{rank}(A) = 2$ Çözüm yok.