4) $\sum_{n=1}^{+\infty} \frac{n!}{(n+1)^n}(x+2)^n$

$$\lim_{n\to+\infty}\left|\frac{\frac{(n+1)!}{(n+2)^{n+1}}(x+2)^{\cancel{n+1}}}{\frac{n!}{(n+1)^n}\cancel{(x+2)^n}}\right| = \lim_{n\to+\infty}\left|\frac{\overset{n+1}{\cancel{(n+1)!}}}{\cancel{n!}}\frac{(n+1)^n}{(n+2)^{n+1}}(x+2)\right|$$

$$= \lim_{n\to+\infty}\frac{(n+1)^{n+1}}{(n+2)^{n+1}}|x+2| = |x+2|\lim_{n\to+\infty}\left(\frac{n+1}{n+2}\right)^{n+1} \overset{1^\infty}{=} |x+2|\,e^{-1} < 1$$

$\Rightarrow |x+2| < e$

$\Rightarrow -e < x+2 < e$

$\Rightarrow -2-e < x < -2+e$

$$\lim_{n\to+\infty}\left(\frac{n+1}{n+2}\right)^{n+1} \overset{1^\infty}{=} e^{-1} = \frac{1}{e}$$

Y.M $= -2$

Y.Y $= e$

Y.A $= (-2-e, -2+e)$

$$\lim_{n\to+\infty}(n+1)\ln\left(\frac{n+1}{n+2}\right) \overset{0.\infty}{=} \lim_{n\to+\infty}\frac{\ln\left(\frac{n+1}{n+2}\right)}{\frac{1}{n+1}}$$

$$\overset{0/0}{=} \lim_{n\to+\infty}\frac{\frac{\overbrace{1\cdot(n+2)-1(n+1)}^{1}}{(n+2)^2}\cdot\frac{1}{\frac{n+1}{n+2}}}{-\frac{1}{(n+1)^2}} = \lim_{n\to+\infty}\frac{-(n+1)^{\cancel{2}}}{(n+2)^{\cancel{2}}}\cdot\frac{\cancel{n+1}}{\cancel{n+1}} = -1$$

$x=-2-e \Rightarrow \sum_{n=1}^{+\infty}\frac{n!}{(n+1)^n}(\cancel{-2}-e\cancel{+2})^n = \sum_{n=1}(-1)^n\cdot\frac{n!}{(n+1)^n}\cdot e^n$???

$x=-2+e \Rightarrow \sum_{n=1}^{+\infty}\frac{n!}{(n+1)^n}(\cancel{-2}+e\cancel{+2})^n = \sum_{n=1}^{+\infty}\frac{n!}{(n+1)^n}e^n$???

5) $\sum_{n=0}^{+\infty} \frac{(-1)^n x^n}{n!}$

$$\lim_{n\to+\infty} \left| \frac{\frac{\cancel{(-1)^{n+1}}\ \cancel{x^{n+1}}}{(n+1)!}}{\frac{\cancel{(-1)^n}\ \cancel{x^n}}{n!}} \right| = \lim_{n\to+\infty} \left| (-1) \frac{\cancel{n!}}{\cancel{(n+1)!}_{\,n+1}} x \right|$$

$$= \lim_{n\to+\infty} \frac{1}{n+1} |x| = 0.|x| < 1$$

Y.M $= 0$

Y.Y $= +\infty$

Y.A $= (-\infty, +\infty)$

6) $\sum_{n=1}^{+\infty} \frac{n!}{2^n} (x-3)^n$

$$\lim_{n\to+\infty} \left| \frac{\frac{\cancel{(n+1)!}^{\,n+1}}{\cancel{2^{n+1}}} \cancel{(x-3)^{n+1}}}{\frac{\cancel{n!}}{\cancel{2^n}} \cancel{(x-3)^n}} \right| = \lim_{n\to+\infty} \left| \frac{n+1}{2} (x-3) \right|$$

$$= \lim_{n\to+\infty} \frac{n+1}{2} |x-3| = \begin{cases} +\infty & , \quad x \neq 3 \\ 0 & , \quad x = 3 \end{cases} \quad \begin{matrix} < 1 \\ ? \end{matrix}$$

Y.M $= 3$

Y.Y $= 0$

Y.A $= \{3\}$

7) $\sum_{n=1}^{+\infty} \frac{2.4.6.\ldots.2n}{3.5.7\ldots(2n-1)} x^{4n}$

$$\lim_{n\to+\infty} \left| \frac{\frac{\cancel{2.4.6}\ldots\cancel{2n}.(2n+2)}{\cancel{3.5.7}\ldots\cancel{(2n-1)}(2n+1)} x^{4(n+1)}}{\frac{\cancel{2.4.6}\ldots\cancel{2n}}{\cancel{3.5.7}\ldots\cancel{(2n-1)}} \cancel{x^{4n}}} \right| = \lim_{n\to+\infty} \left| \frac{2n+2}{2n+1} x^4 \right|$$

$$= \lim_{n\to+\infty} \underbrace{\frac{2n+2}{2n+1}}_{1} |x|^4 = |x|^4 < 1$$

$$\Rightarrow |x| < \sqrt[4]{1} = 1$$

$$\Rightarrow -1 < x < 1$$

Y.M. = 0

Y.Y. = 1

Y.A. = $(-1, 1)$

$x = -1$
$x = 1$ $\Rightarrow \sum_{n=1}^{+\infty} \frac{2.4.6 \ldots 2n}{3.5.7 \ldots (2n-1)}$ ıraksaktır.

$\forall n \in \mathbb{N}$ için $1 < 2 \cdot \frac{4}{3} \cdot \frac{6}{5} \cdots \frac{2n}{2n-1}$

$\sum_{n=1}^{+\infty} 1$ serisi G.T.T. ıraksaktır. Karşılaştırma T.

$\sum_{n=1}^{+\infty} 2 \cdot \frac{4}{3} \cdot \frac{6}{5} \cdots \frac{2n}{2n-1}$ ıraksaktır.

Alıştırmalar:

Aşağıdaki kuvvet serilerinin Y.M., Y.Y., Y.A. ?

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n} x^n, \quad \sum_{n=1}^{+\infty} \frac{5^n}{n!} (x+2)^n, \quad \sum_{n=1}^{+\infty} \frac{(4x-5)^n}{3^n}$$

$$\sum_{n=2}^{+\infty} \frac{(-1)^n (x-1)^n}{n \ln n}, \quad \sum_{n=1}^{+\infty} \frac{3^n}{(-2)^n n(n+1)} (x+5)^n, \quad \sum_{n=1}^{+\infty} \frac{n^2}{3^{2n}} (x+7)^n$$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{9^n} (x-2)^{2n+1}, \quad \sum_{n=0}^{+\infty} \frac{5^n}{(2n)!} \left(\frac{x+1}{3}\right)^n, \quad \sum_{n=1}^{+\infty} \left(\frac{n}{n+1}\right)^{n^2} (x-e)^n$$

$$\sum_{n=0}^{+\infty} \left(\frac{x^2+2}{6}\right)^{n^2}$$