# Matrisler ve Lineer Denklem sistemleri

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$1 \le i \le m$, $1 \le j \le n$ için $a_{ij} \in \mathbb{R}$ ise $A$'ya $m$ satır $n$ sütunlu ($m\times n$ tipinde) reel matris denir.

## Matrislerin Türleri, Özellikleri, İşlemleri:

1) $A=[a_{ij}]_{m\times n}$ ve $A=B \Rightarrow B=[b_{ij}]_{m\times n}$ ve $1\le i\le m$, $1\le j\le n$ için $a_{ij}=b_{ij}$ 'dir.

2) $A=[a_{ij}]_{m\times n} \Rightarrow A^T=[a_{ji}]_{n\times m}$ matrisine $A$'nın transpozesi denir.

$$A=\begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & -3 \end{bmatrix}_{2\times 3} \Rightarrow A^T=\begin{bmatrix} 1 & 0 \\ 0 & 4 \\ -2 & -3 \end{bmatrix}_{3\times 2}$$

3) $O_{m\times n}=[0]_{m\times n}$ matrisine $m\times n$ tipinde sıfır matrisi denir.

$$O=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 5} \qquad O=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{3\times 2}$$

4) $A=[a_{ij}]_{m\times 1}$ sütun matrisi $B=[b_{ij}]_{1\times n}$ satır matrisi denir

$A=\begin{bmatrix}1\\2\\4\\0\end{bmatrix}_{4\times 1}$ sütun $B=\begin{bmatrix}1 & -1 & 0 & 0{,}79 & \frac{1}{e} & \ln 2\end{bmatrix}_{1\times 6}$

satır matrisi

5) $A=[a_{ij}]_{m\times n}$ için $m=n$ ise $A$'ya kare matris denir $A=[a_{ij}]_{n\times n}$

6) $I=[\Delta_{ij}]_{n\times n}$ için $\Delta_{ij}=\begin{cases}1 & , i=j \text{ ise}\\ 0 & , i\neq j \text{ ise}\end{cases}$

matrisine $n\times n$ tipinde birim matris denir.

$I=[1]_{1\times 1}$, $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}_{2\times 2}$, $I=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}_{3\times 3}$ ...

7) $k\in\mathbb{R}$, $A=[a_{ij}]_{m\times n}$ için

$k.A=k[a_{ij}]_{m\times n}=[k a_{ij}]_{m\times n}$ skalerle çarpma

$$3\begin{bmatrix}2 & 1\\0 & -2\\4 & 3\end{bmatrix}=\begin{bmatrix}6 & 3\\0 & -6\\12 & 9\end{bmatrix}$$

8) $A=[a_{ij}]_{m\times n}$, $B=[b_{ij}]_{m\times n}$

$A+B=[a_{ij}]_{m\times n}+[b_{ij}]_{m\times n}=[a_{ij}+b_{ij}]_{m\times n}$ toplama

$$\begin{bmatrix}1 & 0 & -2\\0 & 1 & 4\end{bmatrix}_{2\times 3}+\begin{bmatrix}0 & 0 & 1 & 7\\0 & 1 & 4 & -3\end{bmatrix}_{2\times 4}=\text{tanımsız}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 4 & 8 \\ 5 & 2 & -4 \end{bmatrix} = \begin{bmatrix} -6 & 4 & 6 \\ 5 & 3 & 0 \end{bmatrix}$$

9) $A = [a_{ij}]_{m \times n}$ , $B = [b_{jk}]_{n \times \ell}$ için

$$A.B = [a_{ij}]_{m \times n} \cdot [b_{jk}]_{n \times \ell} = [c_{ik}]_{m \times \ell}$$

çarpma

$$c_{ik} = a_{i1} b_{1k} + a_{i2} b_{2k} + \ldots\ldots + a_{in} b_{nk}$$

$$\underset{3 \times 2}{\begin{bmatrix} 1 & 2 \\ 4 & 3 \\ 0 & 1 \end{bmatrix}} \cdot \underset{2 \times 4}{\begin{bmatrix} 0 & 1 & 3 & 2 \\ 4 & -1 & 0 & 4 \end{bmatrix}} = \underset{3 \times 4}{\begin{bmatrix} 8 & -1 & 3 & 10 \\ 12 & 1 & 12 & 20 \\ 4 & -1 & 0 & 4 \end{bmatrix}}$$

10) $A_{n \times n}$ matrisi verilsin. Eğer

$$A_{n \times n} \cdot B_{n \times n} = B_{n \times n} \cdot A_{n \times n} = I_{n \times n}$$

olacak şekilde bir $B_{n \times n}$ matrisi mevcutsa $B$'ye $A$'nın tersi denir. $B = A^{-1}$ ile gösterilir.

$$\begin{bmatrix} 2 & 7 & 0 \\ 1 & 4 & -1 \\ 2 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} -24 & 35 & 7 \\ 7 & -10 & -2 \\ 4 & -6 & -1 \end{bmatrix} = \begin{bmatrix} -24 & 35 & 7 \\ 7 & -10 & -2 \\ 4 & -6 & -1 \end{bmatrix} \begin{bmatrix} 2 & 7 & 0 \\ 1 & 4 & -1 \\ 2 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Determinantlar:

a) $A = [a_{11}]_{1\times 1} \Rightarrow \det(A) = |A| = |a_{11}| = a_{11}$

b) $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2} \Rightarrow$

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$\begin{vmatrix} 2 & 1 \\ 4 & 7 \end{vmatrix} = 2.7 - 4.1 = 14 - 4 = 10$$

c) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

1.yol.

$$\det(A) = |\Delta| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

2.yol. (sarus kuralı)

$$\det(A) = |\Delta| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{21} a_{32} a_{13} + a_{13} a_{12} a_{23}$$

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix} \qquad - a_{21} a_{12} a_{33} - a_{11} a_{32} a_{23} - a_{31} a_{22} a_{13}$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 2 & 1 & -4 \\ 7 & 1 & 3 \\ 1 & 2 & 0 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} - 1 \cdot \begin{vmatrix} 7 & 3 \\ 1 & 0 \end{vmatrix} + (-4)\begin{vmatrix} 7 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 2 \cdot (-6) - 1 \cdot (-3) + (-4) \cdot 13 = -61$$

$$\begin{array}{|ccc|} 2 & 1 & -4 \\ 7 & 1 & 3 \\ 1 & 2 & 0 \\ \hline \end{array} = 0 + (-56) + 3 - 0 - 12 - (-4) = -61$$
$$\begin{array}{ccc} 2 & 1 & -4 \\ 7 & 1 & 3 \end{array}$$

$$\begin{array}{cccc} + & - & + & - \end{array}$$
$$\begin{vmatrix} 2 & 0 & -3 & 1 \\ 4 & 1 & 7 & 3 \\ 2 & -8 & 4 & -1 \\ 4 & 1 & 3 & -2 \end{vmatrix} = 2\begin{vmatrix} 1 & 7 & 3 \\ -8 & 4 & -1 \\ 1 & 3 & -2 \end{vmatrix} - 0 \cdot \begin{vmatrix} 4 & 7 & 3 \\ 2 & 4 & -1 \\ 4 & 3 & -2 \end{vmatrix}$$
$$+ (-3)\begin{vmatrix} 4 & 1 & 3 \\ 2 & -8 & -1 \\ 4 & 1 & -2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 4 & 1 & 7 \\ 2 & -8 & 4 \\ 4 & 1 & 3 \end{vmatrix}$$