

Örnekler:

1) a) $\begin{cases} x+y=1 \\ 2x+2y=4 \end{cases}$ b) $\begin{cases} 3x-y=1 \\ 2x+y=4 \end{cases}$ c) $\begin{cases} 2x+4y=4 \\ x+2y=2 \end{cases}$ L.D.S. çözüm.

Çözüm:

a) $\begin{bmatrix} 1 & 1 & | & 1 \\ 2 & 2 & | & 4 \end{bmatrix} \xrightarrow{-2S_1+S_2 \rightarrow S_2} \begin{bmatrix} 1 & 1 & | & 1 \\ 0 & 0 & | & 2 \end{bmatrix} \xrightarrow{\frac{1}{2}S_2 \rightarrow S_2} \begin{bmatrix} 1 & 1 & | & 1 \\ 0 & 0 & | & 1 \end{bmatrix}$

$\xrightarrow{-S_2+S_1 \rightarrow S_1} \begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 1 \end{bmatrix}$

$\mathcal{K} = \emptyset$

$\text{rank}[A:B] = 2 > 1 = \text{rank}(A) \Rightarrow$ Çözüm yok.

b) $\begin{bmatrix} 3 & -1 & | & 1 \\ 2 & 1 & | & 4 \end{bmatrix} \xrightarrow{-S_2+S_1 \rightarrow S_1} \begin{bmatrix} 1 & -2 & | & -3 \\ 2 & 1 & | & 4 \end{bmatrix} \xrightarrow{-2S_1+S_2 \rightarrow S_2} \begin{bmatrix} 1 & -2 & | & -3 \\ 0 & 5 & | & 10 \end{bmatrix}$

$\xrightarrow{\frac{1}{5}S_2 \rightarrow S_2} \begin{bmatrix} 1 & -2 & | & -3 \\ 0 & 1 & | & 2 \end{bmatrix} \xrightarrow{2S_2+S_1 \rightarrow S_1} \begin{bmatrix} 1 & 0 & | & 1 \\ 0 & 1 & | & 2 \end{bmatrix}$

$\mathcal{K} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

$\text{rank}[A:B] = 2 = \text{rank}(A) = n \Rightarrow$ Tek çözüm var

$\begin{cases} x=1 \\ y=2 \end{cases} \Rightarrow X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$x=1$
 $y=2$ tek çözüm

c) $\begin{bmatrix} 2 & 4 & | & 4 \\ 1 & 2 & | & 2 \end{bmatrix} \xrightarrow{S_1 \leftrightarrow S_2} \begin{bmatrix} 1 & 2 & | & 2 \\ 2 & 4 & | & 4 \end{bmatrix} \xrightarrow{-2S_1+S_2 \rightarrow S_2} \begin{bmatrix} 1 & 2 & | & 2 \\ 0 & 0 & | & 0 \end{bmatrix}$

$\text{rank}[A:B] = 1 = \text{rank}(A) < 2 = n$

$2-1=1$ değişkene
bağıli sonsuz çözüm var.

$$\begin{cases} x + 2y = 2 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x = 2 - 2y \\ y = y \end{cases}$$

$$C_K = \left\{ \begin{bmatrix} 2-2y \\ y \end{bmatrix} \mid y \in \mathbb{R} \right\}$$

$$2) \begin{cases} 3x - 5y + 3z = 4 \\ x - 2y - z = 1 \\ 2x + 3y - 5z = -1 \end{cases} \Rightarrow C_K = ?$$

$$\begin{bmatrix} 3 & -5 & 3 & | & 4 \\ 1 & -2 & -1 & | & 1 \\ 2 & 3 & -5 & | & -1 \end{bmatrix} \xrightarrow{S_1 \leftrightarrow S_2} \begin{bmatrix} 1 & -2 & -1 & | & 1 \\ 3 & -5 & 3 & | & 4 \\ 2 & 3 & -5 & | & -1 \end{bmatrix}$$

$$\begin{matrix} -3S_1 + S_2 \rightarrow S_2 \\ -2S_1 + S_3 \rightarrow S_3 \end{matrix} \begin{bmatrix} 1 & -2 & -1 & | & 1 \\ 0 & 1 & 6 & | & 1 \\ 0 & 7 & -3 & | & -3 \end{bmatrix} \xrightarrow{-7S_2 + S_3 \rightarrow S_3} \begin{bmatrix} 1 & -2 & -1 & | & 1 \\ 0 & 1 & 6 & | & 1 \\ 0 & 0 & -45 & | & -10 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{-45}S_3 \rightarrow S_3 \\ -6S_2 + S_2 \rightarrow S_2 \\ S_3 + S_1 \rightarrow S_1 \end{matrix} \begin{bmatrix} 1 & -2 & -1 & | & 1 \\ 0 & 1 & 6 & | & 1 \\ 0 & 0 & 1 & | & \frac{2}{9} \end{bmatrix} \xrightarrow{\begin{matrix} -6S_2 + S_2 \rightarrow S_2 \\ S_3 + S_1 \rightarrow S_1 \end{matrix}} \begin{bmatrix} 1 & -2 & 0 & | & \frac{11}{9} \\ 0 & 1 & 0 & | & -\frac{1}{3} \\ 0 & 0 & 1 & | & \frac{2}{9} \end{bmatrix}$$

$$\begin{matrix} 2S_2 + S_1 \rightarrow S_1 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & | & \frac{5}{9} \\ 0 & 1 & 0 & | & -\frac{1}{3} \\ 0 & 0 & 1 & | & \frac{2}{9} \end{bmatrix}$$

$$\text{rank}[A:B] = 3 = \text{rank}(A) = n$$

Teş Cözüm

$$C_K = \left\{ \begin{bmatrix} \frac{5}{9} \\ -\frac{1}{3} \\ \frac{2}{9} \end{bmatrix} \right\}$$

$$3) \begin{cases} x - 2y + z = 0 \\ 3x - t = 4 \\ y + 3z = 1 \\ y + t - z = -1 \end{cases}$$

L.D.S. çözünüz

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 4 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{array} \right] \xrightarrow{-3S_1 + S_2 \rightarrow S_2} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 6 & -3 & -1 & 4 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{array} \right]$$

$$\xrightarrow{S_3 \leftrightarrow S_2} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 6 & -3 & -1 & 4 \\ 0 & 1 & -1 & 1 & -1 \end{array} \right] \xrightarrow{\begin{matrix} -6S_2 + S_3 \rightarrow S_3 \\ -S_2 + S_4 \rightarrow S_4 \end{matrix}} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & -21 & -1 & -2 \\ 0 & 0 & -4 & 1 & -2 \end{array} \right]$$

$$\xrightarrow{-S_3 \rightarrow S_3} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 21 & 1 & 2 \\ 0 & 0 & -4 & 1 & -2 \end{array} \right] \xrightarrow{5S_4 + S_3 \rightarrow S_3} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 6 & -8 \\ 0 & 0 & -4 & 1 & -2 \end{array} \right]$$

$$\xrightarrow{4S_3 + S_4 \rightarrow S_4} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 6 & -8 \\ 0 & 0 & 0 & 25 & -34 \end{array} \right]$$

$\text{rank}[A:B] = 4 = \text{rank}(A) = n$
tek çözüm

$$z = -8 + \frac{204}{25} = \frac{4}{25}$$

$$t = -\frac{34}{25}$$

$$z + 6t = -8$$

$$z - \frac{6 \cdot 34}{25} = -8$$

$$y + 3z = 1$$

$$y + \frac{12}{25} = 1$$

$$y = -\frac{13}{25}$$

$$x - 2y + z = 0$$

$$x + \frac{26}{25} + \frac{4}{25} = 0$$

$$\Rightarrow \boxed{x = -\frac{30}{25}}$$

$$\Rightarrow \mathcal{K} = \left\{ \begin{bmatrix} -\frac{30}{25} \\ -\frac{13}{25} \\ \frac{4}{25} \\ -\frac{34}{25} \end{bmatrix} \right\}$$