   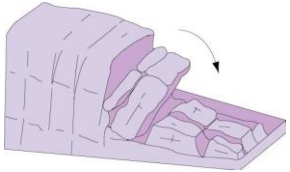

# MINE 1000 DYNAMICS

## 2019 – 2020 Spring

## Week 3 (Kinematics of a particle)


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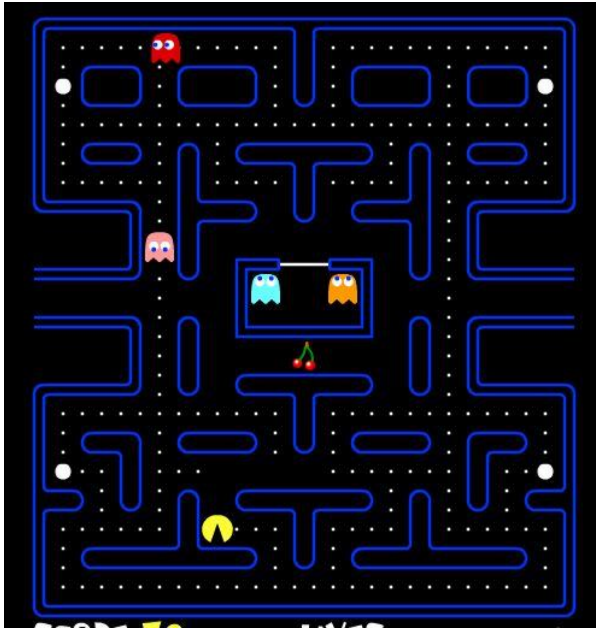
# Course Outline

| Week | Date | Course Content |
|---|---|---|
| 1 | | Warming up, general introduction to dynamics |
| 2 | | Kinematics of a particle |
| 3 | | Kinematics of a particle |
| 4 | | Kinetics of a particle: Force & acceleration |
| 5 | | Kinetics of a particle: Work & energy |
| 6 | | Kinetics of a particle: Impulse & momentum |
| 7 | | General review & problem solving |
| 8 | | Midterm exam week |
| 9 | | Kinematics of a rigid body |
| 10 | | Kinematics of a rigid body |
| 11 | | Kinetics of a rigid body: Force & acceleration |
| 12 | | Kinetics of a rigid body: Work & energy |
| 13 | | Kinetics of a rigid body: Impulse & momentum |
| 14 | | General review & problem solving |
| 15 | | Final exam week |

# Kinematics

## Kinematics

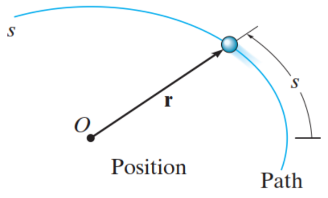
### Rectilinear motion

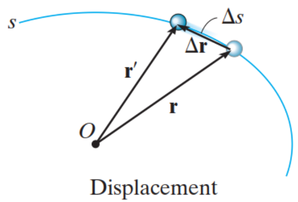


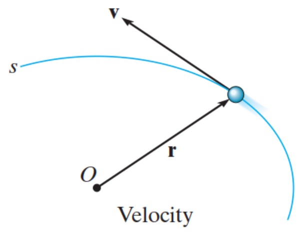
### Curvilinear motion

# Kinematics / Curvilinear Motion

**Curvilinear motion occurs when a particle moves along a curved path. Since this path is often described in three dimensions, vector analysis will be used to formulate the particle's position, velocity, and acceleration.**




$$\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta t}$$

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt}}$$

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt}}$$

# Kinematics / Curvilinear Motion

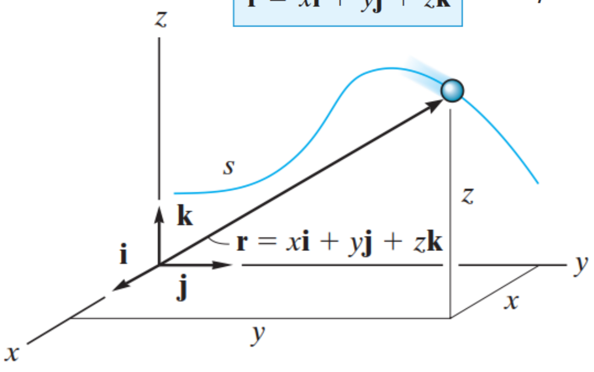
## Rectangular Components:

**Occasionally the motion of a particle can best be described along a path that can be expressed in terms of its x, y, z coordinates.**

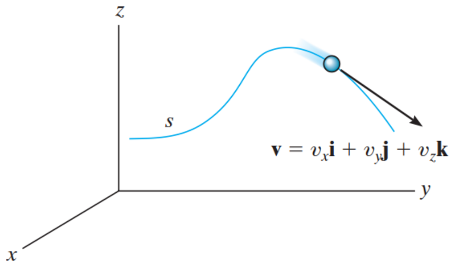
$$\boxed{\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$




Position




Velocity

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}}$$

where

$$\boxed{v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}}$$

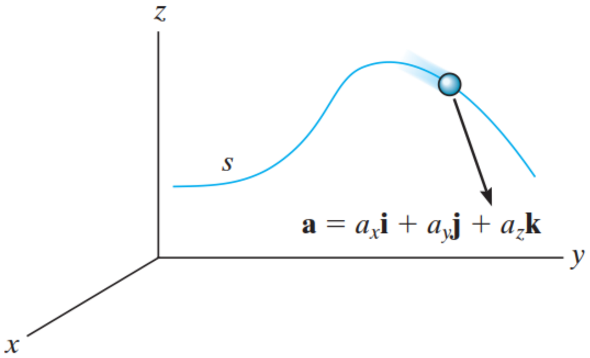
# Kinematics / Curvilinear Motion

## Rectangular Components:

The "dot" notation $\dot{x}$, $\dot{y}$, $\dot{z}$ represents the first time derivatives of $x = x(t)$, $y = y(t)$, $z = z(t)$, respectively.

The velocity has a *magnitude* that is found from

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$




Acceleration

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\boxed{\begin{aligned} a_x &= \dot{v}_x = \ddot{x} \\ a_y &= \dot{v}_y = \ddot{y} \\ a_z &= \dot{v}_z = \ddot{z} \end{aligned}}$$

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}}$$

# Kinematics / Curvilinear Motion

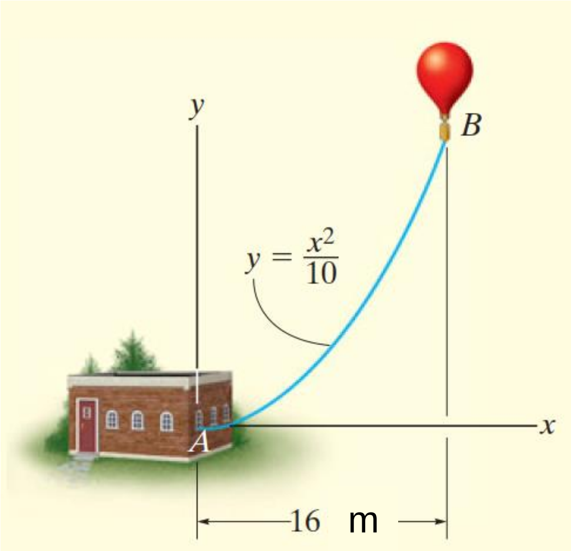
## Rectangular Components:

### Important Points

- Curvilinear motion can cause changes in *both* the magnitude and direction of the position, velocity, and acceleration vectors.
- The velocity vector is always directed *tangent* to the path.
- In general, the acceleration vector is *not* tangent to the path, but rather, it is tangent to the hodograph.
- If the motion is described using rectangular coordinates, then the components along each of the axes do not change direction, only their magnitude and sense (algebraic sign) will change.
- By considering the component motions, the change in magnitude and direction of the particle's position and velocity are automatically taken into account.

## Numerical Example:

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (8t)$ m where $t$ is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



# Kinematics / Curvilinear Motion

## Numerical Example:

**Velocity.** The velocity component in the $x$ direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ m/s}$$

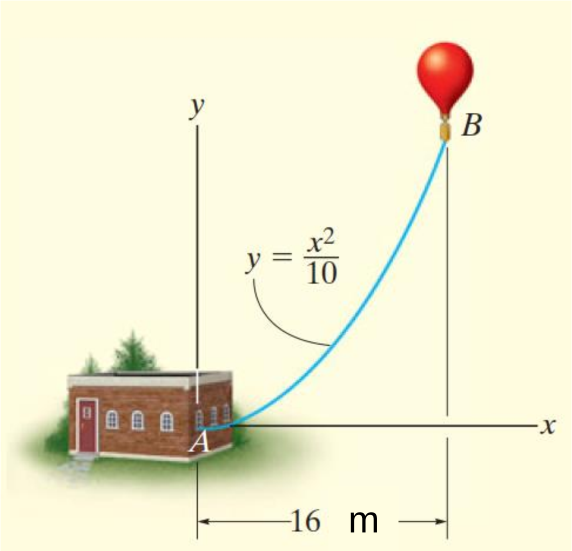
$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ m/s} \uparrow$$

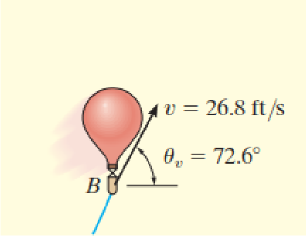
When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ m/s})^2 + (25.6 \text{ m/s})^2} = 26.8 \text{ m/s}$$

The direction is tangent to the path, Fig. 12–18$b$, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6^\circ$$



## Numerical Example:

**Acceleration.** The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

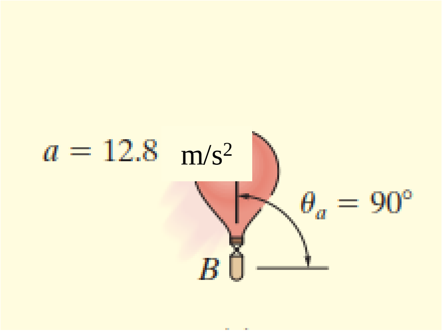
$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ m/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ m/s}^2$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90°$$



# Kinematics / Curvilinear Motion

## Numerical Example:

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

# Kinematics / Curvilinear Motion

## Numerical Example:

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, since $v_y = 10$ m/s, then

$$y = v_y t; \qquad 100 \text{ m} = (10 \text{ m/s})\, t \qquad t = 10 \text{ s}$$

**Velocity.** Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

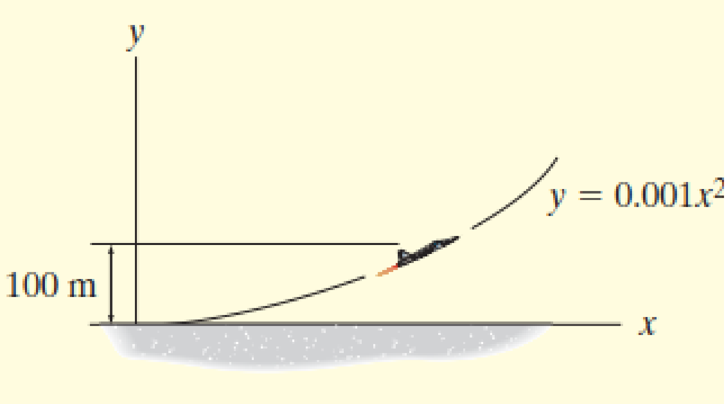
$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x$$

Thus

$$10 \text{ m/s} = 0.002(316.2 \text{ m})(v_x)$$
$$v_x = 15.81 \text{ m/s}$$

The magnitude of the velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81 \text{ m/s})^2 + (10 \text{ m/s})^2} = 18.7 \text{ m/s}$$



# Kinematics / Curvilinear Motion

## Numerical Example:

**Acceleration.** Using the chain rule, the time derivative of gives the relation between the acceleration components.

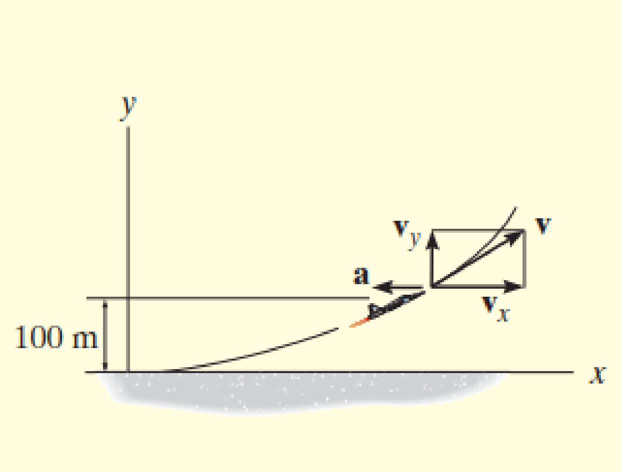
$$a_y = \dot{v}_y = 0.002\dot{x}v_x + 0.002x\dot{v}_x = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002((15.81 \text{ m/s})^2 + 316.2 \text{ m}(a_x))$$
$$a_x = -0.791 \text{ m/s}^2$$

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791 \text{ m/s}^2)^2 + (0 \text{ m/s}^2)^2}$$
$$= 0.791 \text{ m/s}^2$$



# Kinematics / Curvilinear Motion

## Moton of a Projectile

**The free-flight motion of a projectile is often studied in terms of its rectangular components.**

**Horizontal Motion.** Since $a_x = 0$, application of the constant acceleration equations, 12–4 to 12–6, yields

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t; \qquad v_x = (v_0)_x$$

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad x = x_0 + (v_0)_x t$$

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(x - x_0); \qquad v_x = (v_0)_x$$

The first and last equations indicate that *the horizontal component of velocity always remains constant during the motion.*

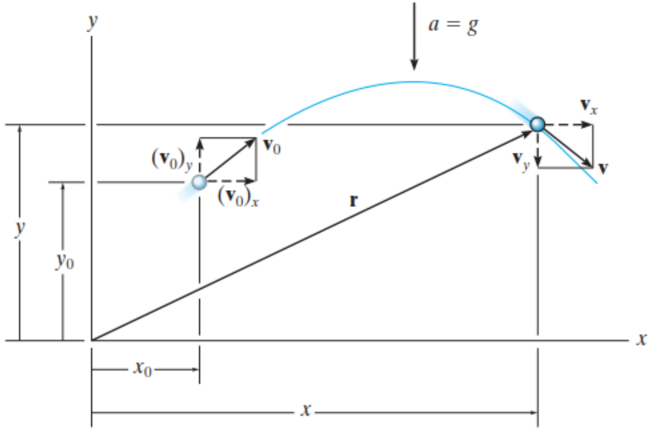
**Vertical Motion.** Since the positive $y$ axis is directed upward, then $a_y = -g$. Applying Eqs. 12–4 to 12–6, we get

$$(+\uparrow) \qquad v = v_0 + a_c t; \qquad v_y = (v_0)_y - gt$$

$$(+\uparrow) \qquad y = y_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$

$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(y - y_0); \qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

Recall that the last equation can be formulated on the basis of eliminating the time $t$ from the first two equations, and therefore *only two of the above three equations are independent of one another.*




To summarize, problems involving the motion of a projectile can have at most three unknowns since only three independent equations can be written; that is, *one* equation in the *horizontal direction* and *two* in the *vertical direction*. Once $\mathbf{v}_x$ and $\mathbf{v}_y$ are obtained, the resultant velocity $\mathbf{v}$, which is *always tangent* to the path, can be determined by the *vector sum*

# Kinematics / Curvilinear Motion

## Moton of a Projectile

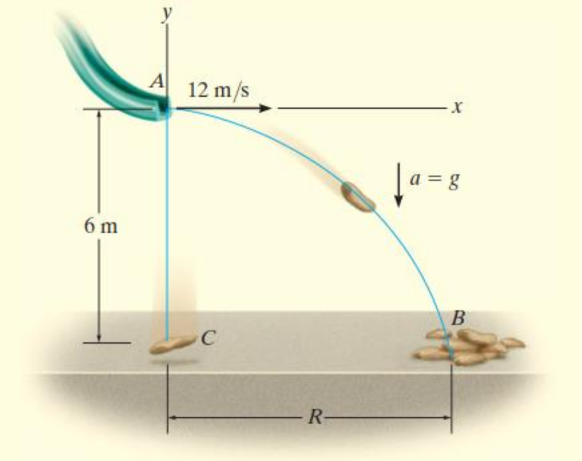
To summarize, problems involving the motion of a projectile can have at most three unknowns since only three independent equations can be written; that is, *one* equation in the *horizontal direction* and *two* in the *vertical direction*. Once $\mathbf{v}_x$ and $\mathbf{v}_y$ are obtained, the resultant velocity $\mathbf{v}$, which is *always tangent* to the path, can be determined by the *vector sum*

Gravel falling off the end of this conveyor belt follows a path that can be predicted using the equations of constant acceleration. In this way the location of the accumulated pile can be determined. Rectangular coordinates are used for the analysis since the acceleration is only in the vertical direction.

# Kinematics / Curvilinear Motion

**Numerical Example:**

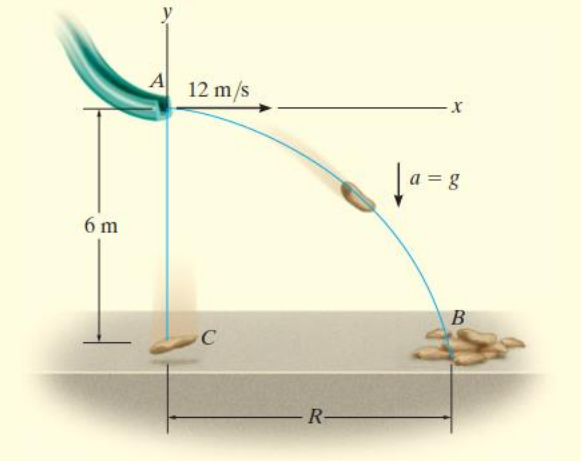
A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range $R$ where sacks begin to pile up.



# Kinematics / Curvilinear Motion

## Numerical Example:

**Coordinate System.** The origin of coordinates is established at the beginning of the path, point $A$, Fig. 12–21. The initial velocity of a sack has components $(v_A)_x = 12$ m/s and $(v_A)_y = 0$. Also, between points $A$ and $B$ the acceleration is $a_y = -9.81$ m/s$^2$. Since $(v_B)_x = (v_A)_x = 12$ m/s, the three unknowns are $(v_B)_y$, $R$, and the time of flight $t_{AB}$. Here we do not need to determine $(v_B)_y$.




**Vertical Motion.** The vertical distance from $A$ to $B$ is known, and therefore we can obtain a direct solution for $t_{AB}$ by using the equation

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$

$$-6\text{ m} = 0 + 0 + \tfrac{1}{2}(-9.81\text{ m/s}^2)t_{AB}^2$$

$$t_{AB} = 1.11\text{ s} \qquad \textit{Ans.}$$

**Horizontal Motion.** Since $t_{AB}$ has been calculated, $R$ is determined as follows:

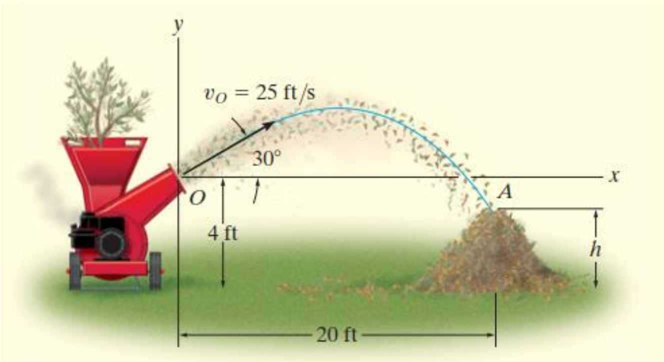
$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t_{AB}$$

$$R = 0 + 12\text{ m/s}\,(1.11\text{ s})$$

$$R = 13.3\text{ m} \qquad \textit{Ans.}$$

# Kinematics / Curvilinear Motion

## Numerical Example:




**Coordinate System.** When the motion is analyzed between points $O$ and $A$, the three unknowns are the height $h$, time of flight $t_{OA}$, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at $O$, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (25 \cos 30°) \text{ ft/s} = 21.65 \text{ ft/s} \rightarrow$$

$$(v_O)_y = (25 \sin 30°) \text{ ft/s} = 12.5 \text{ ft/s} \uparrow$$

Also, $(v_A)_x = (v_O)_x = 21.65$ ft/s and $a_y = -32.2$ ft/s$^2$. Since we do not need to determine $(v_A)_y$, we have

**Horizontal Motion.**

$$(\overset{+}{\rightarrow}) \qquad x_A = x_O + (v_O)_x t_{OA}$$

$$20 \text{ ft} = 0 + (21.65 \text{ ft/s}) t_{OA}$$

$$t_{OA} = 0.9238 \text{ s}$$

**Vertical Motion.** Relating $t_{OA}$ to the initial and final elevations of a chip, we have

$$(+\uparrow) \qquad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2} a_c t_{OA}^2$$

$$(h - 4 \text{ ft}) = 0 + (12.5 \text{ ft/s})(0.9238 \text{ s}) + \tfrac{1}{2}(-32.2 \text{ ft/s}^2)(0.9238 \text{ s})^2$$

$$h = 1.81 \text{ ft} \qquad \textit{Ans.}$$

# Kinematics / Curvilinear Motion

## In-Class Application 1: Do it yourself!!! You have 10 min.

The position of a particle is $\mathbf{r} = \{(3t^3 - 2t)\mathbf{i} - (4t^{1/2} + t)\mathbf{j} + (3t^2 - 2)\mathbf{k}\}$ m, where $t$ is in seconds. Determine the magnitude of the particle's velocity and acceleration when $t = 2$ s.

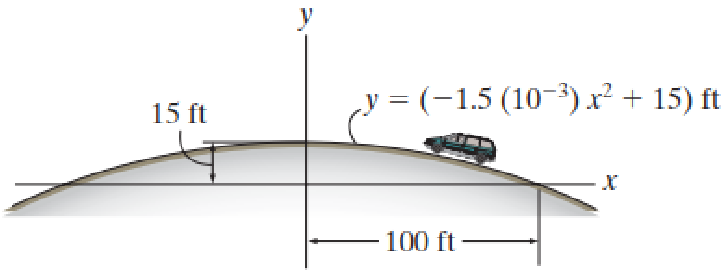
## In-Class Application 2: Do it yourself!!! You have 10 min.

A particle travels along the parabolic path $y = bx^2$. If its component of velocity along the $y$ axis is $v_y = ct^2$, determine the $x$ and $y$ components of the particle's acceleration. Here $b$ and $c$ are constants.

# Kinematics / Curvilinear Motion

## In-Class Application 3: Do it yourself!!! You have 10 min.

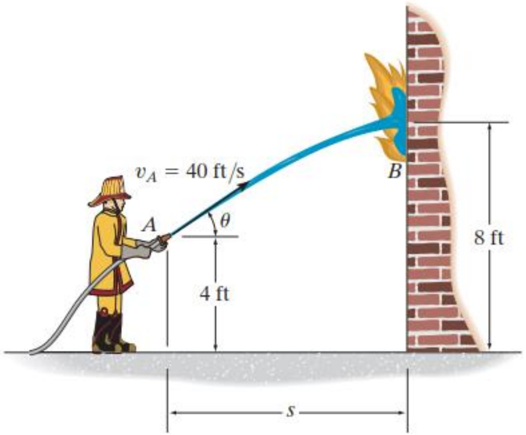
The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)\,\text{ft}$. If it has a constant speed of $75\,\text{ft/s}$, determine the $x$ and $y$ components of the van's velocity and acceleration when $x = 50\,\text{ft}$.



# Kinematics / Curvilinear Motion

## In-Class Application 4: Do it yourself!!! You have 10 min.

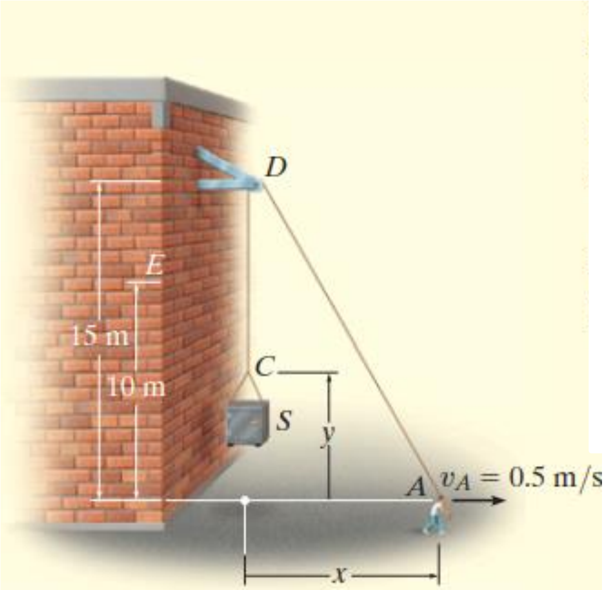
The fireman holds the hose at an angle $\theta = 30°$ with horizontal, and the water is discharged from the hose at $A$ with a speed of $v_A = 40 \text{ ft/s}$. If the water stream strikes the building at $B$, determine his two possible distances $s$ from the building.



# Kinematics / Absolute Dependent Motion

## Numerical Example:

A man at $A$ is hoisting a safe $S$ as shown in Fig. 12–41 by walking to the right with a constant velocity $v_A = 0.5$ m/s. Determine the velocity and acceleration of the safe when it reaches the elevation of 10 m. The rope is 30 m long and passes over a small pulley at $D$.



# Kinematics / Absolute Dependent Motion

## Numerical Example:

**Position-Coordinate Equation.** This problem is unlike the previous examples since rope segment $DA$ changes *both direction and magnitude*. However, the ends of the rope, which define the positions of $S$ and $A$, are specified by means of the $x$ and $y$ coordinates since they must be measured from a fixed point and *directed along the paths of motion* of the ends of the rope.

The $x$ and $y$ coordinates may be related since the rope has a fixed length $l = 30$ m, which at all times is equal to the length of segment $DA$ plus $CD$. Using the Pythagorean theorem to determine $l_{DA}$, we have $l_{DA} = \sqrt{(15)^2 + x^2}$; also, $l_{CD} = 15 - y$. Hence,

$$l = l_{DA} + l_{CD}$$

/s

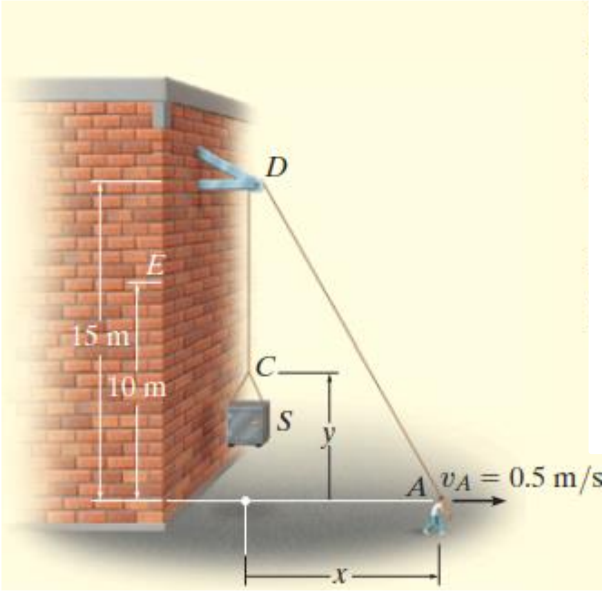
$$30 = \sqrt{(15)^2 + x^2} + (15 - y)$$

$$y = \sqrt{225 + x^2} - 15 \qquad (1)$$

**Time Derivatives.** Taking the time derivative, using the chain rule (see Appendix C), where $v_S = dy/dt$ and $v_A = dx/dt$, yields

$$v_S = \frac{dy}{dt} = \left[\frac{1}{2}\frac{2x}{\sqrt{225 + x^2}}\right]\frac{dx}{dt}$$

$$= \frac{x}{\sqrt{225 + x^2}}v_A \qquad (2)$$



# Kinematics / Absolute Dependent Motion

## Numerical Example:

At $y = 10$ m, $x$ is determined from Eq. 1, i.e., $x = 20$ m. Hence, from Eq. 2 with $v_A = 0.5$ m/s,

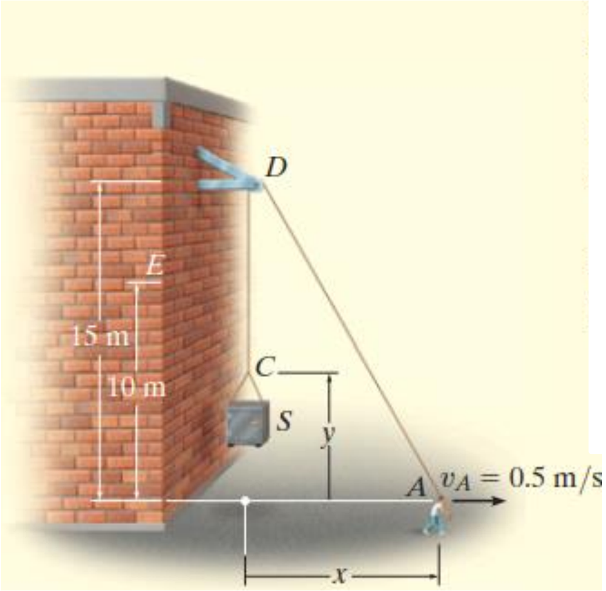
$$v_S = \frac{20}{\sqrt{225 + (20)^2}}(0.5) = 0.4\text{m/s} = 400\,\text{mm/s} \uparrow \quad \textit{Ans.}$$

The acceleration is determined by taking the time derivative of Eq. 2. Since $v_A$ is constant, then $a_A = dv_A/dt = 0$, and we have

$$a_S = \frac{d^2y}{dt^2} = \left[\frac{-x(dx/dt)}{(225 + x^2)^{3/2}}\right]xv_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]\left(\frac{dx}{dt}\right)v_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]x\frac{dv_A}{dt} = \frac{225v_A^2}{(225 + x^2)^{3/2}}$$

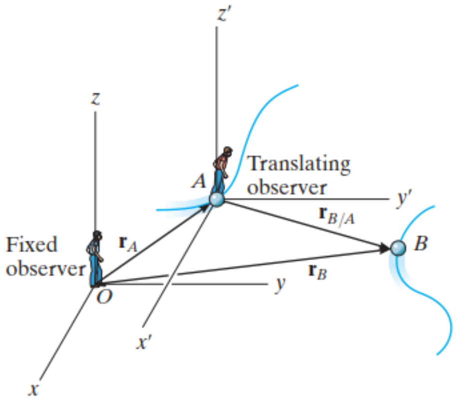
At $x = 20$ m, with $v_A = 0.5$ m/s, the acceleration becomes

$$a_S = \frac{225(0.5\text{ m/s})^2}{[225 + (20\text{ m})^2]^{3/2}} = 0.00360\text{ m/s}^2 = 3.60\text{ mm/s}^2 \uparrow \textit{Ans.}$$



# Kinematics / Relative Motion

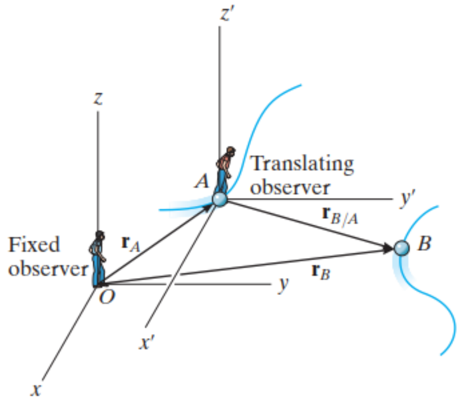
**Throughout this chapter the absolute motion of a particle has been determined using a single fixed reference frame. There are many cases, however, where the path of motion for a particle is complicated, so that it may be easier to analyze the motion in parts by using two or more frames of reference. For example, the motion of a particle located at the tip of an airplane propeller, while the plane is in flight, is more easily described if one observes first the motion of the airplane from a fixed reference and then superimposes (vectorially) the circular motion of the particle measured from a reference attached to the airplane.**



# Kinematics / Relative Motion

**Position.** Consider particles $A$ and $B$, which move along the arbitrary paths shown in Fig. 12–42. The *absolute position* of each particle, $\mathbf{r}_A$ and $\mathbf{r}_B$, is measured from the common origin $O$ of the *fixed* $x$, $y$, $z$ reference frame. The origin of a second frame of reference $x'$, $y'$, $z'$ is attached to and moves with particle $A$. The axes of this frame are *only permitted to translate* relative to the fixed frame. The position of $B$ measured relative to $A$ is denoted by the *relative-position vector* $\mathbf{r}_{B/A}$. Using vector addition, the three vectors shown in Fig. 12–42 can be related by the equation

$$\boxed{\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}}$$



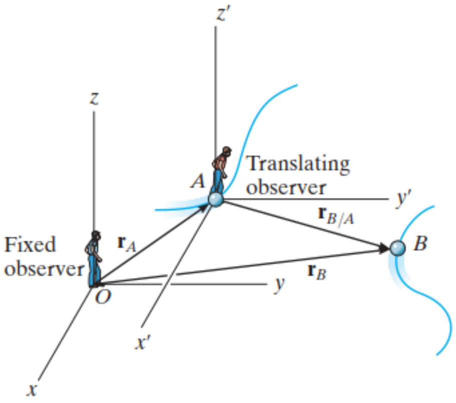

**Velocity.** An equation that relates the velocities of the particles is determined by taking the time derivative of the above equation; i.e.,

$$\boxed{\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}}$$

# Kinematics / Relative Motion

**Acceleration.** The time derivative of Eq. 12–34 yields a similar vector relation between the *absolute* and *relative accelerations* of particles $A$ and $B$.

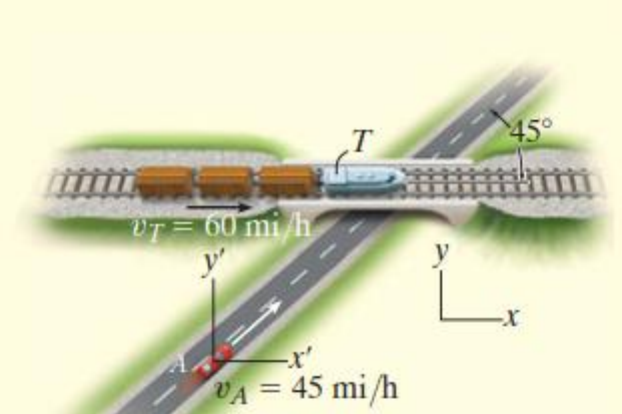
$$\boxed{\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}}$$



# Kinematics / Relative Motion

## Numerical Example:

A train travels at a constant speed of 60 mi/h, crosses over a road as shown in Fig. 12–43*a*. If the automobile *A* is traveling at 45 mi/h along the road, determine the magnitude and direction of the velocity of the train relative to the automobile.



(a)

## Numerical Example:

**Vector Analysis.** The relative velocity $\mathbf{v}_{T/A}$ is measured from the translating $x'$, $y'$ axes attached to the automobile, Fig. 12–43*a*. It is determined from $\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$. Since $\mathbf{v}_T$ and $\mathbf{v}_A$ are known in *both* magnitude and direction, the unknowns become the $x$ and $y$ components of $\mathbf{v}_{T/A}$. Using the $x$, $y$ axes in Fig. 12–43*a*, we have

$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$60\mathbf{i} = (45\cos 45°\mathbf{i} + 45\sin 45°\mathbf{j}) + \mathbf{v}_{T/A}$$

$$\mathbf{v}_{T/A} = \{28.2\mathbf{i} - 31.8\mathbf{j}\}\ \text{mi/h} \qquad \textit{Ans.}$$

The magnitude of $\mathbf{v}_{T/A}$ is thus

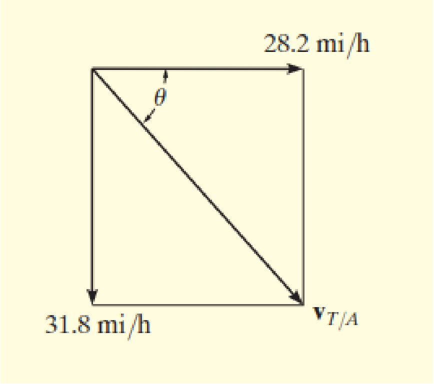
$$v_{T/A} = \sqrt{(28.2)^2 + (-31.8)^2} = 42.5\ \text{mi/h} \qquad \textit{Ans.}$$

From the direction of each component, Fig. 12–43*b*, the direction of $\mathbf{v}_{T/A}$ is

$$\tan\theta = \frac{(v_{T/A})_y}{(v_{T/A})_x} = \frac{31.8}{28.2}$$

$$\theta = 48.5° \qquad \textit{Ans.}$$

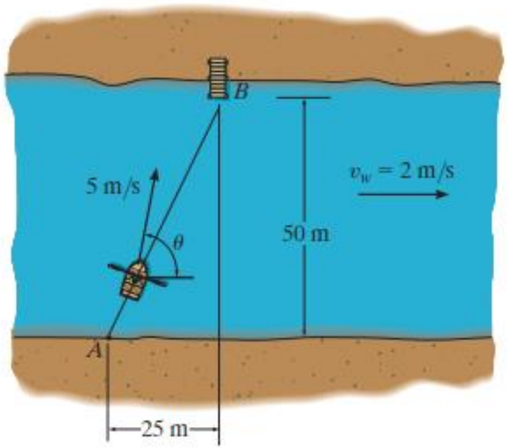
Note that the vector addition shown in Fig. 12–43*b* indicates the correct sense for $\mathbf{v}_{T/A}$. This figure anticipates the answer and can be used to check it.



# Kinematics / Relative Motion

## In-Class Application 7: Do it yourself!!! You have 10 min.

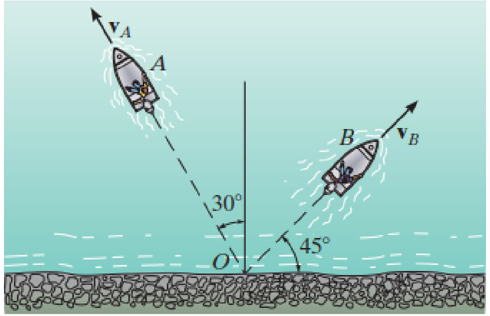
The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle $\theta$ he must direct the boat so that it travels from $A$ to $B$.



# Kinematics / Relative Motion

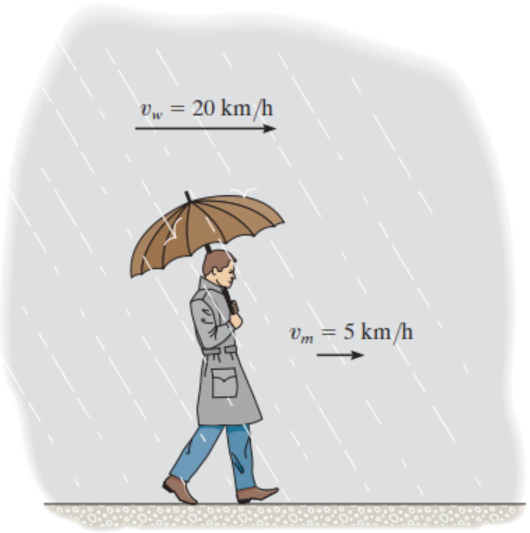
## In-Class Application 8: Do it yourself!!! You have 10 min.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20\,\text{ft/s}$ and $v_B = 15\,\text{ft/s}$, determine the velocity of boat $A$ with respect to boat $B$. How long after leaving the shore will the boats be 800 ft apart?



## In-Class Application 9: Do it yourself!!! You have 10 min.

**12–230.** A man walks at 5 km/h in the direction of a 20-km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine the direction in which the drops appear to fall with respect to the man. Assume the horizontal speed of the raindrops is equal to that of the wind.



# Essential Vocabulary:

Curvilinear motion: Eğrisel hareket

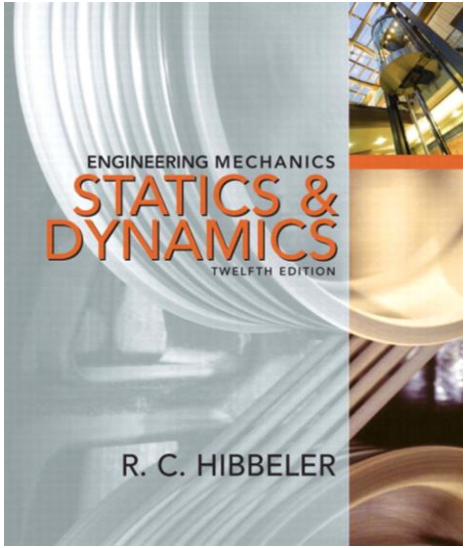
Chain rule: Zincir kuralı

Projectile: Mermi

Datum: Kıyas kotu

Cord: İp, halat

**Course Reference:**



# End of the lecture…

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