


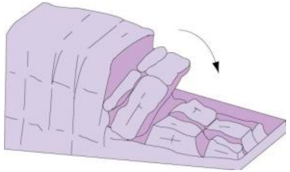

MINE 1000 DYNAMICS

2019 – 2020 Spring

Exercises (Particle Dynamics)
27/04/2020

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Course Outline

Week	Date	Course Content
1		Warming up, general introduction to dynamics
2		Kinematics of a particle
3		Kinematics of a particle
4		Kinetics of a particle: Force & acceleration
5		Kinetics of a particle: Work & energy
6		Kinetics of a particle: Impulse & momentum
7		General review & problem solving
9		Kinematics of a rigid body
10		Kinematics of a rigid body
11		Kinetics of a rigid body: Force & acceleration
12		Kinetics of a rigid body: Work & energy
13		Kinetics of a rigid body: Impulse & momentum
14		General review & problem solving

Exercise 1:

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

Kinematics (rectilinear motion)

Exercise 1:

$$v = v_0 + a_c t$$

$$120 = 70 + (6000)(t) \quad t = 8.33(10^{-3})\,h$$

$$\boxed{= 30\,s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$120^2 = 70^2 + 2(6000)(s - 0)$$

$$s = \underline{0.792\ km}$$

$$\boxed{= 792\ m}$$

Exercise 2:

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

Kinematics (rectilinear motion)

Exercise 2:

$$ds = v\frac{dv}{a}$$

$$\int_0^s ds = \int_6^v \frac{v}{-15v^{1/2}}\,dv \rightarrow s = \int_6^v -0.0667\,v^{1/2}\,dv$$

$$s = -0.0444\,v^{3/2} + 6.532$$

$$v = 0 \rightarrow s = -0.0444\,(0)^{3/2} + 6.532 = \boxed{6.53\,m}$$

Exercise 2:

$$dt = \frac{dv}{a} \longrightarrow \int_0^t dt = -\int_6^v \frac{dv}{1.5\, v^{1/2}}$$

$$t = -1.33 \left(v^{1/2}\right)\Big|_6^v = \left(3.266 - 1.333\, v^{1/2}\right)$$

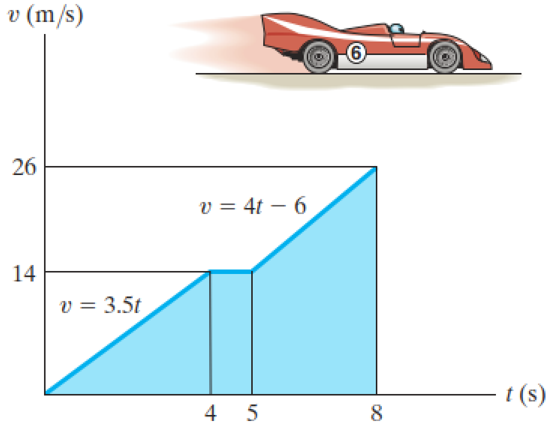
$v = 0$

$$t = \left(3.266 - 1.333 \cdot (0)\right)$$

$$\boxed{t = 3.275}$$

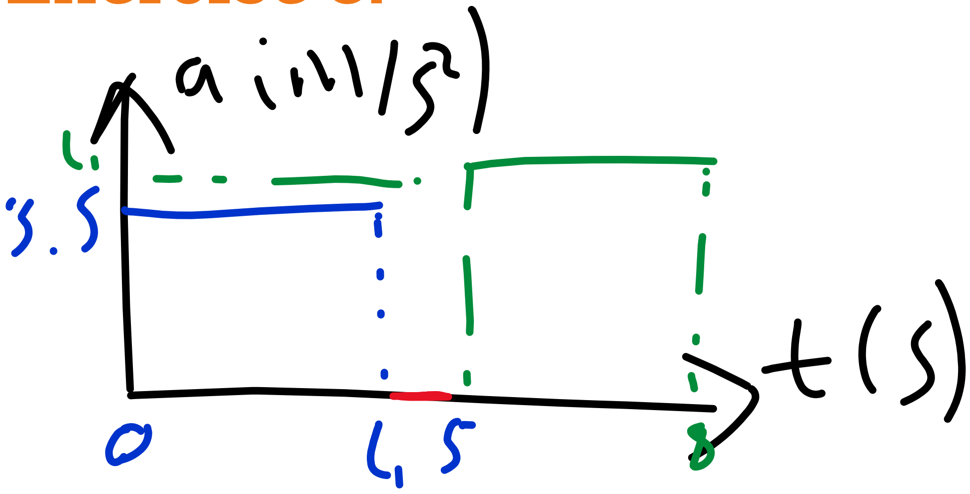
Exercise 3:

The race car starts from rest and travels along a straight road until it reaches a speed of 26 m/s in 8 s as shown on the v–t graph. The flat part of the graph is caused by shifting gears. Draw the a–t graph and determine the maximum acceleration of the car.



Kinematics (rectilinear motion)

Exercise 3:



$0 \leq t < 4\ s$

$$a = \frac{\Delta v}{\Delta t} = \frac{14}{4}$$

$$\boxed{a = 3.5\ m/s^2}$$

$4 \leq t < 5$

$$a = \frac{\Delta v}{\Delta t} = \frac{0}{1} = 0$$

$5 \leq t < 8$

$$a = \frac{\Delta v}{\Delta t} = \frac{26-14}{8-5} = \boxed{4\ m/s^2}$$

Exercise 4:

If the velocity of a particle is defined as $v(t) = 0.8t^2\, \mathbf{i} + 12\, t^{1/2}\, \mathbf{j} + 5\, \mathbf{k}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when t=2s.

Kinematics (curvilinear motion)

Exercise 4:

$$V = \underbrace{0.8t^2}_{x} i + \underbrace{12t^{1/2}}_{y} j + \underbrace{5}_{z} k$$

$$\alpha \qquad \beta \qquad \gamma$$

$$a = \frac{dv}{dt} = 1.6t\, i + 6t^{1/2} j$$

$$t = 2s \rightarrow a = 3.2 i + 4.243 j$$

Exercise 4:

$$\bar{a} = \sqrt{(3.2)^2 + (4.245)^2} = \boxed{5.31\ m/s^2}$$

$$\hat{a} = u_a = \frac{a}{\bar{a}} = \frac{3.2i + 4.245j}{5.31}$$

$$\boxed{\hat{a} = 0.6022i + 0.7984j}$$

$$\alpha = \cos^{-1}(0.6022) = 53°$$

$$\beta = \cos^{-1}(0.7984) = 37°$$

$$\gamma = \cos^{-1}(0) = 90°$$

Exercise 5:

The velocity of a particle is $v = 3\,i + (6-2t)\,j$ m/s, where t is in seconds. If $r = 0$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

Kinematics (curvilinear motion)

Exercise 5:

$$V = 3i + (6 - 2t)j \qquad V = \frac{dr}{dt} \quad dr = v\,dt$$

$$\int_0^r dr = \int_0^t 3i + (6 - 2t)j\,dt$$

$$r = (3t\,i + (6t - t^2)j)\,m$$

Exercise 5:

$t = 1s \rightarrow r_1 = 3i + 5j \text{ m}$

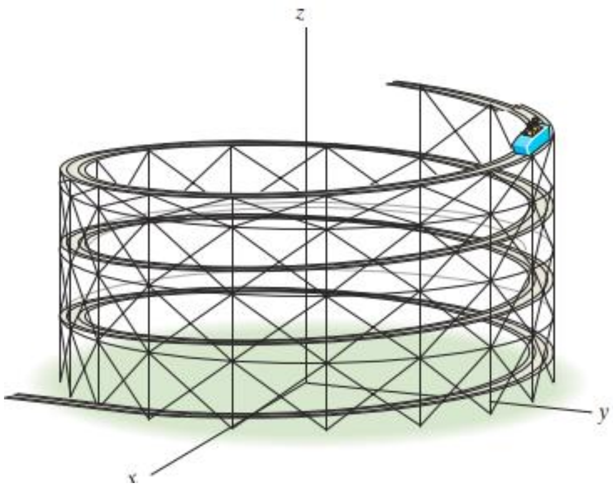
$t = 3s \rightarrow r_3 = 9i + 9j \text{ m}$

$\Delta r = r_3 - r_1 = 9i + 9j - 3i - 5j$

$$\boxed{\Delta r = 6i + 4j \text{ m}}$$

Exercise 6:

The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



Exercise 6:

$x = c \sin kt \qquad \dot{x} = ck \cos kt \qquad \ddot{x} = -ck^2 \sin kt$

$y = c \cos kt \qquad \dot{y} = -ck \sin kt \qquad \ddot{y} = -ck^2 \cos kt$

$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2}$

$= \sqrt{c^2k^2 + b^2}$

Exercise 6:

$x = c\sin kt \qquad \dot{x} = ck\cos kt \qquad \ddot{x} = -ck^2\sin kt$

$y = c\cos kt \qquad \dot{y} = -ck\sin kt \qquad \ddot{y} = -ck^2\cos kt$

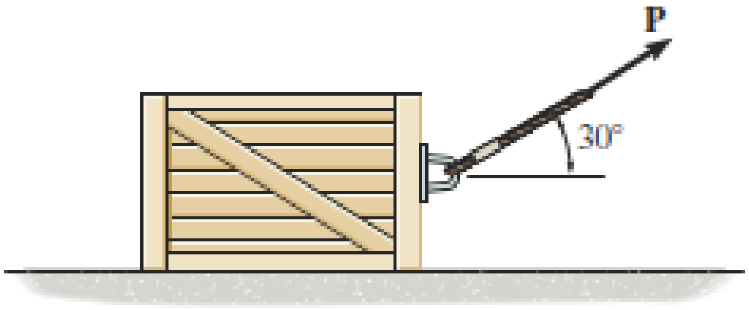
$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

$$a = \sqrt{(ck^2\sin kt)^2 + (-ck^2\cos kt)^2 + 0^2}$$

$$a = ck^2$$

Exercise 7:

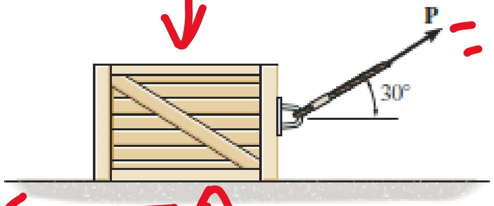
If the coefficient of kinetic friction between the 50-kg crate and the ground is $\mu_k = 0.3$, determine the distance the crate travels and its velocity when t = 3 s. The crate starts from rest, and P = 200 N.



Kinetics (force & acceleration)

Exercise 7:

$50(9.81)$



$= 200N$

$F_f = 0.3N$

N

$\uparrow \sum F_y = m a_y = 0 \qquad N - 50.(9.81) + 200 \sin 30 = 0$

$\boxed{N = 390.5 N}$

$\rightarrow \sum F_x = m a_x \qquad 200 \cos 30 - 0.3(390.5) = 50a$

$\boxed{a = 1.121 \, m/s^2}$

Exercise 7:

$$v = v_0 + a_c t$$

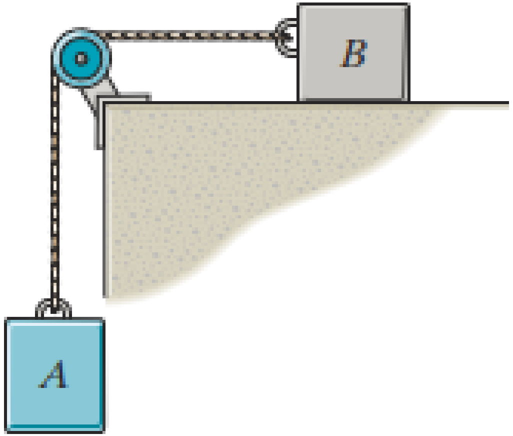
$$v = 0 + 1.121(3) = \boxed{3.36 \text{ m/s}}$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

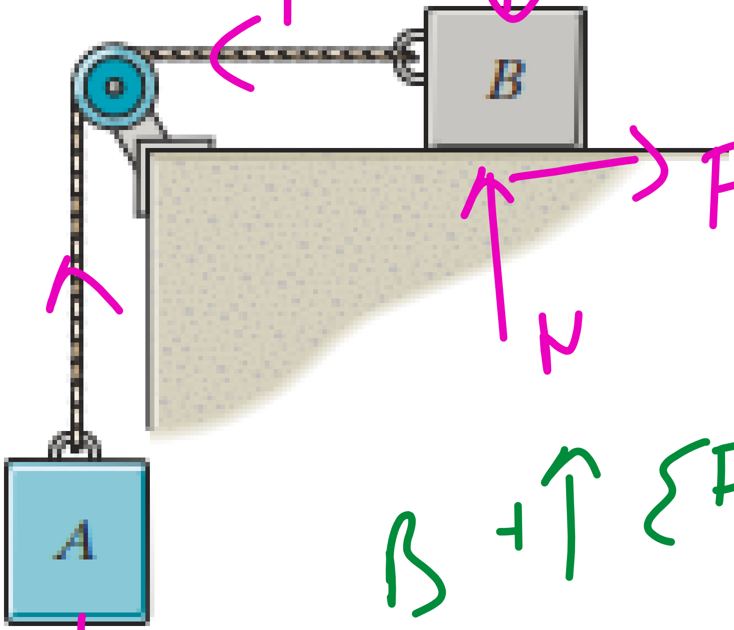
$$s = 0 + 0 + \frac{1}{2}(1.121)(3)^2 = \boxed{5.04 \text{ m}}$$

Exercise 8:

Determine the acceleration of the blocks when the system is released. The coefficient of kinetic friction is μk, and the mass of each block is m. Neglect the mass of the pulleys and cord.



Exercise 8:



$\overleftarrow{a}$ T mg

T a mg

$F_f = \mu_k N$

N

A:

$$+\uparrow \sum F_y = ma$$

$$T - mg = -ma$$

B:

$$+\uparrow \sum F_y = 0 \qquad N - mg = 0$$

$$N = mg$$

$$\overset{+}{\leftarrow} \sum F_x = ma \qquad T - \mu_k mg = ma$$

Exercise 8:

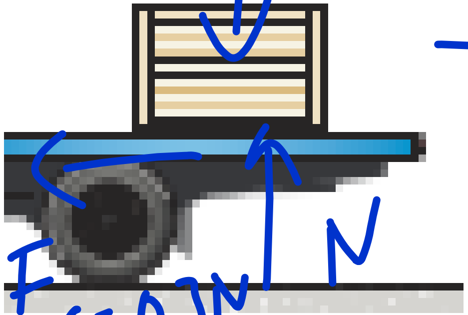
$$a = \frac{1}{2}(1 - \mu_k) g$$

$$T = \frac{1}{2}(1 + \mu_k) mg$$

Exercise 9:

The coefficient of static friction between the 200-kg crate and the flat bed of the truck is μk = 0.3. Determine the shortest time for the truck to reach a speed of 60 km h, starting from rest with constant acceleration, so that the crate does not slip.

Exercise 9:



$+\uparrow \sum F_y = 0$

$N - 200(9.81) = 0$

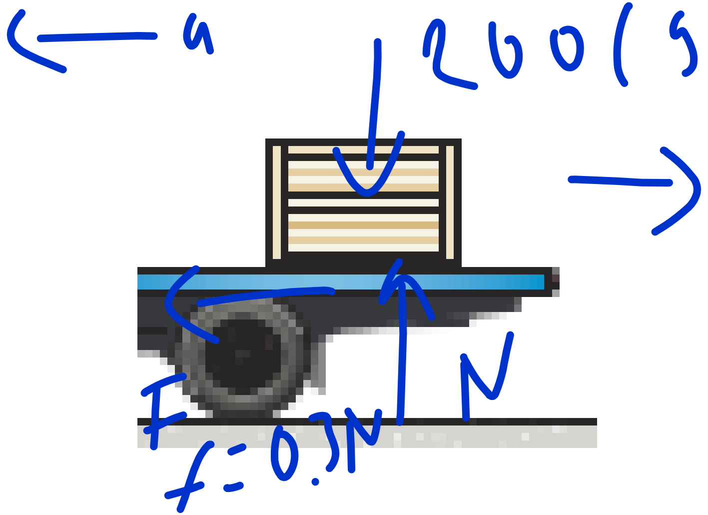
$\boxed{N = 1962\ N}$

$\overset{+}{\rightarrow} \sum F_x = m a_x$

$-0.3(1962) = 200(-a)$

$\boxed{a = 2.943\ m/s^2}$

Exercise 9:



$$V = V_0 + a_c t$$

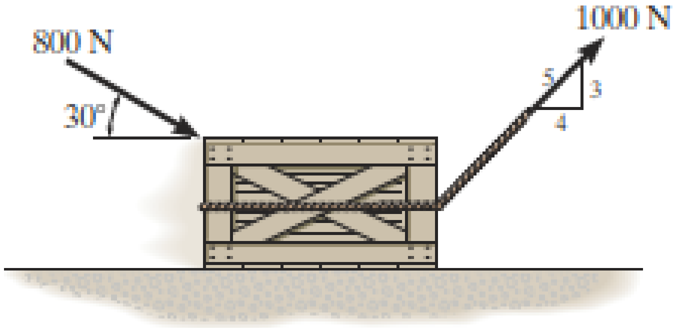
$$16.67 = 0 + 2.94t$$

$$\boxed{t = 5.66\,s}$$

$$V = 60\,km/h = 16.67\,m/s$$

Exercise 10:

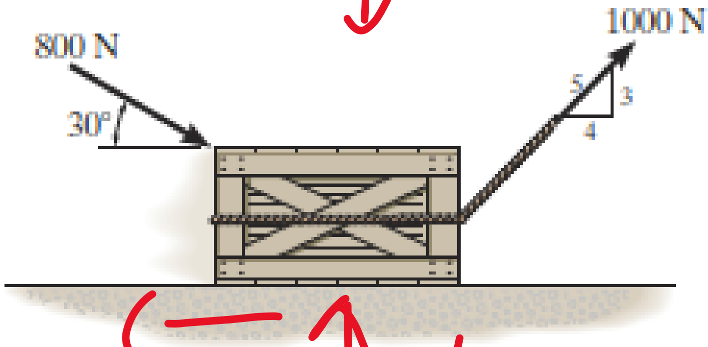
The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is μk = 0.2.



Kinetics (work & energy)

Exercise 10:

$100(9.81)$



$\rightarrow a, v$

$F_f = 0.2N$ $\quad N$

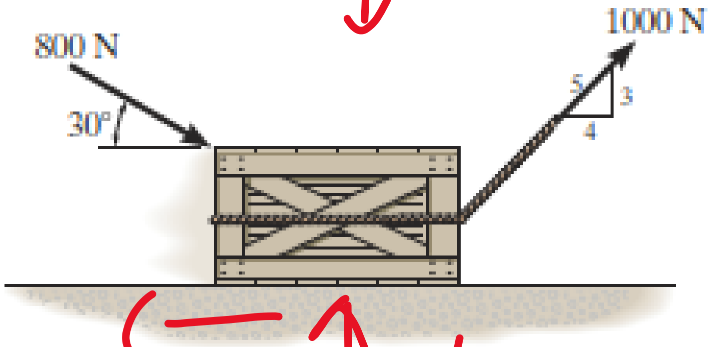
$$+\uparrow \Sigma F_y = 0 \quad N + 1000\left(\tfrac{3}{5}\right) - 800\sin 30 - 981 = 0$$

$$\boxed{N = 781\ \text{N}}$$

Exercise 10:

$\boxed{s = 13.5\ m}$

$100(9.81)$



$\rightarrow a, v$

$F_f = 0.2N$

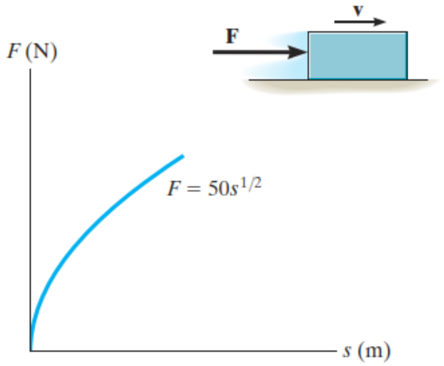
N

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 800\cos 30(s) + 1000\left(\tfrac{4}{5}\right)(s) - 151.2(s) = \tfrac{1}{2}\,100\,(v)^2$$

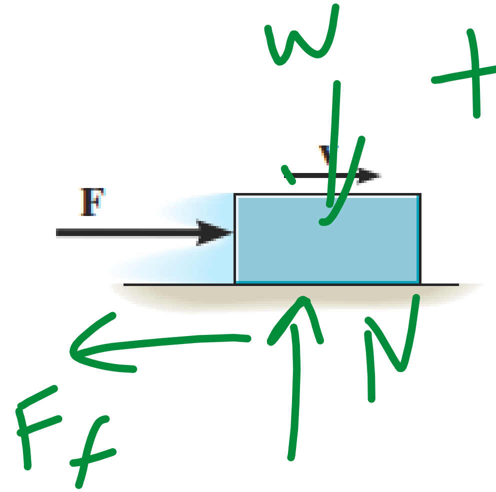
Exercise 11:

The force F, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block must slide before its velocity becomes 15 m/s. When s = 0 the block is moving to the right at v = 6 m/s. The coefficient of kinetic friction between the block and surface μk = 0.3.



Kinetics (work & energy)

Exercise 11:



$$+\uparrow \sum F_y = 0$$

$$N - W = 0$$

$$N = W$$

$$N = 20(9.81)$$

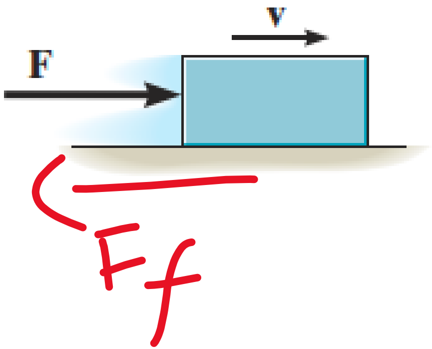
$$\boxed{= 196.2\ N}$$

$$F_f = 0.3N$$

$$= 0.3(196.2)N$$

$$\boxed{= 58.86\ N}$$

Exercise 11:



$$U_F = \int F \cdot ds$$

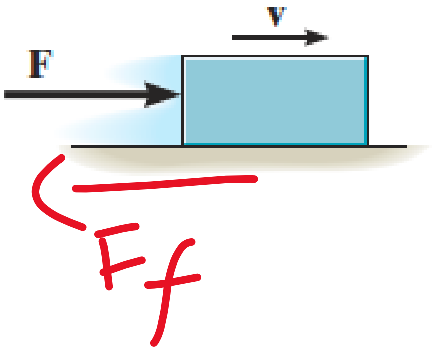
$$= \int_0^s 50 s^{1/2} ds$$

$$= \frac{100}{3} s^{3/2}$$

$$U_{F_f} = F_f \cdot s$$

$$= -58.86 s$$

Exercise 11:



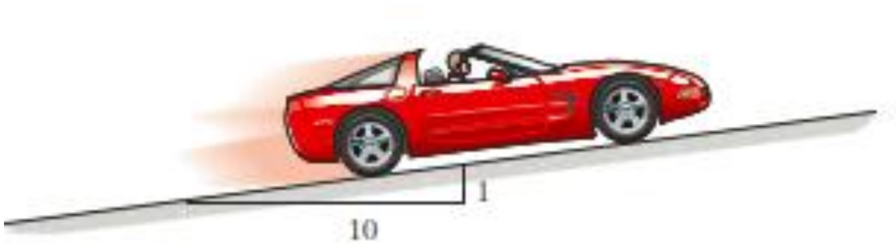
$$T_1 + \sum U_{1\to 2} = T_2$$

$$\frac{1}{2}20(6^2) + \frac{100}{3}s^{3/2} - 58.86s = \frac{1}{2}20(15^2)$$

$$\boxed{\frac{100}{3}s^{3/2} - 58.86s - 1890 = 0} \to \boxed{s = 20.52\,m}$$

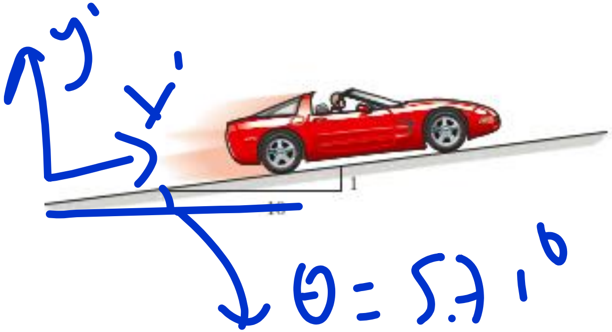
Exercise 12:

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of ε=0.8 .



Kinetics (work & energy)

Exercise 12:



$\theta = 5.71°$

$v = v_0 + a_c t$

$25 = 0 + a_c(30)$

$\boxed{a_c = 0.833 \ m/s^2}$

$\sum F_{x'} = m a_{x'}$

$F - 2000(9.81)\sin 5.71 = 2000(0.833)$

$\boxed{F = 3618.53 \ N}$

Exercise 12:

$$P_{out(max)} = F v$$

$$= 3618.53 \cdot 25$$

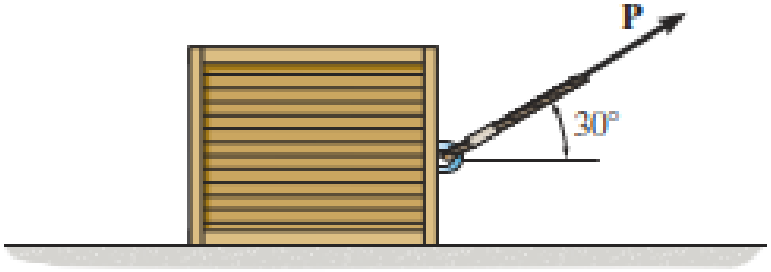
$$\boxed{= 90473.24\,W}$$

$$\varepsilon = \frac{P_{out}}{P_{in}} \Rightarrow 0.8 = \frac{90473.24}{P_{in}}$$

$$P_{in} = 113091.55\,W = \boxed{113\,kW}$$

Exercise 13:

The 50-kg crate is pulled by the constant force P. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of P. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



Kinetics (impulse & momentum)

Exercise 13:

$$F_f = \mu_k N = \boxed{0.2N}$$

$$+\uparrow \quad m v_{1y} + \Sigma \int F_y \, dt = m v_{2y}$$

$$0 + N \cdot 5 + P(5) \cdot \sin 30 - 50(9.81)(5) = 0$$

$$\boxed{N = 490.5 - 0.5P}$$

Exercise 13:

$\xrightarrow{+}$ $mv_{1x} + \sum \int F_x\, dt = mv_{2x}$

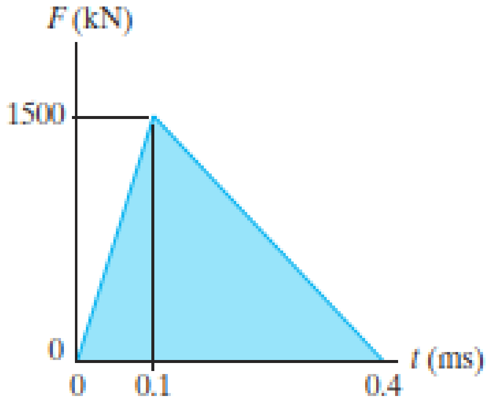
$0 + P(5)(0.5\sqrt{3}) - 0.2N(5) = 50(10)$

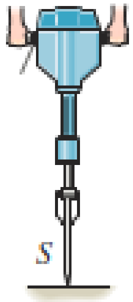
$4.33P - N = 500$

$\boxed{N = 387.5\ N}$ $\boxed{P = 205\ N}$

Exercise 14:

During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike S is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.





Kinetics (impulse & momentum)

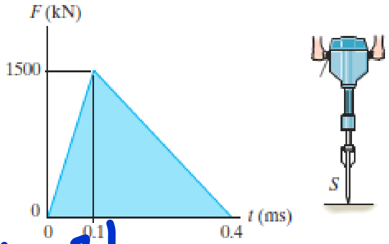
Exercise 14:

$$m(v_y)_1 + \sum \int F_y \, dt = m(v_y)_2$$

$+\uparrow$

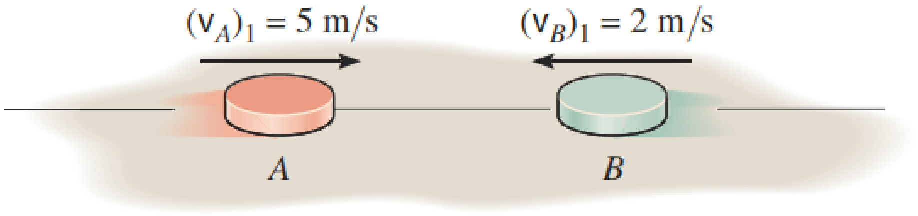
$$2(-90) + \frac{1}{2}\, 0.4(10^{-3})(1500)(10^{3}) = 2v$$

$$\boxed{v = 60 \text{ m/s}}$$



Exercise 15:

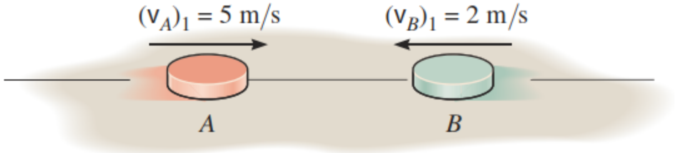
Disk A has a mass of 2 kg and is sliding forward on the smooth surface with a velocity v_{A1} = 5 m/s when it strikes the 4-kg disk B, which is sliding towards A at v_{B1} = 2 m/s with direct central impact. If the coefficient of restitution between the disks is e =0.4, compute the velocities of A and B just after collision.



Kinetics (impulse & momentum)

Exercise 15:

$\xrightarrow{+}$



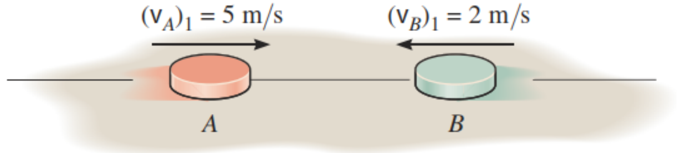
$$m_A (v_{A1}) + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$$

$$2(5) + 4(-2) = 2v_{A2} + 4v_{B2}$$

$$2 = 2v_{A2} + 4v_{B2}$$

$$\boxed{v_{A2} + 2v_{B2} = 1}$$

Exercise 15:



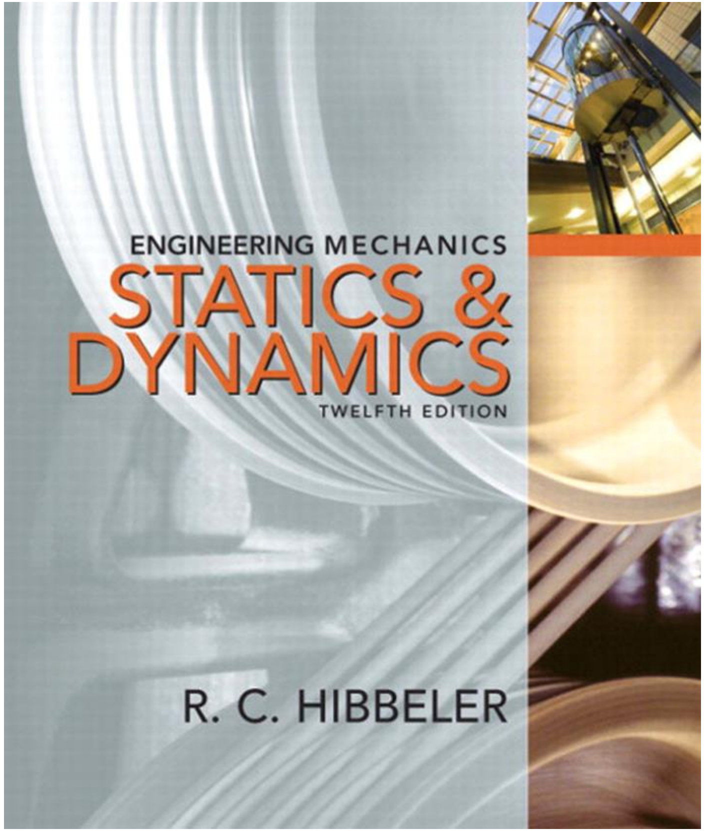
$$e = \frac{V_{B2} - V_{A2}}{V_{A1} - V_{B1}}$$

$$0.4 = \frac{V_{B2} - V_{A2}}{5 - (-2)}$$

$$\boxed{V_{B2} - V_{A2} = 2.8}$$

$$\boxed{\begin{aligned} V_{A2} &= -1.53 \text{ m/s} \\ V_{B2} &= 1.27 \text{ m/s} \end{aligned}}$$

Course Reference:



End of the lecture…

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