Karadeniz Teknik Üniversitesi

Engineering Faculty
Mining Engineering Department

MINE1000-DYNAMICS

Week 4:
Kinetics of a Particle: Force and Acceleration

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Course Content

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Kinetics: Force & Acceleration

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change.The basis for kinetics is Newton's second law, which states that when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force.

$$\mathbf{F} = m\mathbf{a}$$

The above equation, which is referred to as the equation of motion, is one of the most important formulations in mechanics.

Kinetics: Force & Acceleration

Newton's Law of Gravitational Attraction

Shortly after formulating his three laws of motion, Newton postulated a law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as

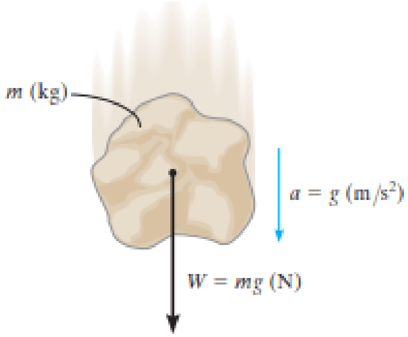
$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

F = force of attraction between the two particles

G = universal constant of gravitation; according to experimental evidence $G = 66.73(10^{-12})\ \mathrm{m^3/(kg \cdot s^2)}$

m_1, m_2 = mass of each of the two particles

r = distance between the centers of the two particles

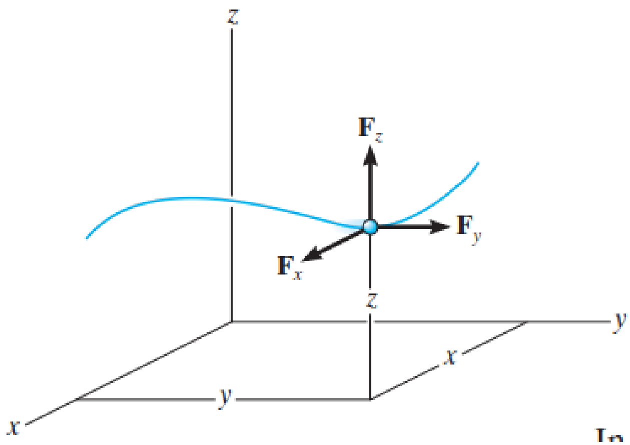


$$\boxed{W = mg\ (\mathrm{N}) \qquad (g = 9.81\ \mathrm{m/s^2})}$$

Kinetics: Force & Acceleration

Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial x, y, z frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their i, j, k components



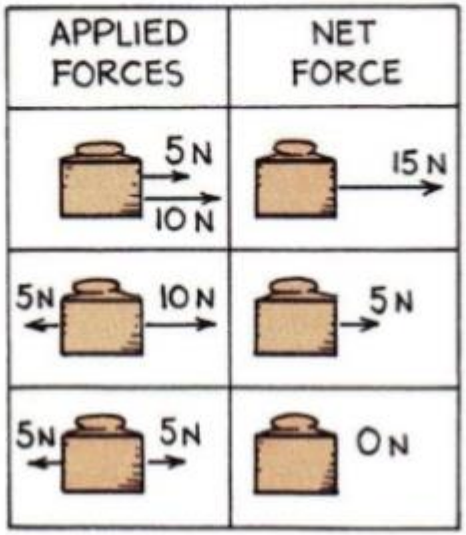
$$\Sigma \mathbf{F} = m\mathbf{a}; \quad \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})$$

$$\Sigma F_x = m a_x$$
$$\Sigma F_y = m a_y$$
$$\Sigma F_z = m a_z$$

Kinetics: Force & Acceleration

Force Causes Acceleration

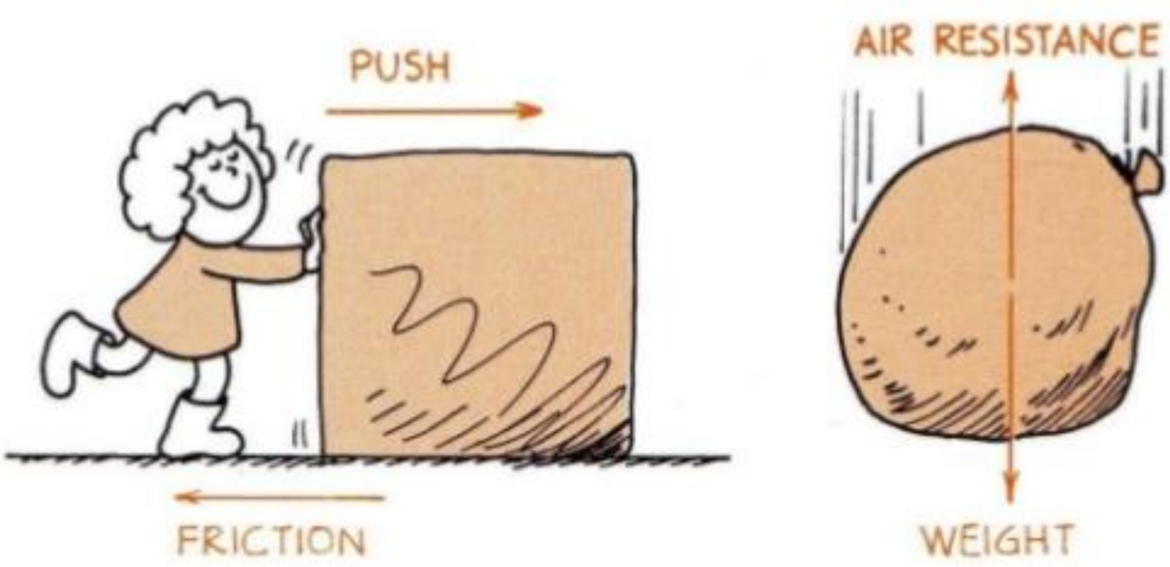
When an objects accelerate, it can be caused by a number of different forces.



Kinetics: Force & Acceleration

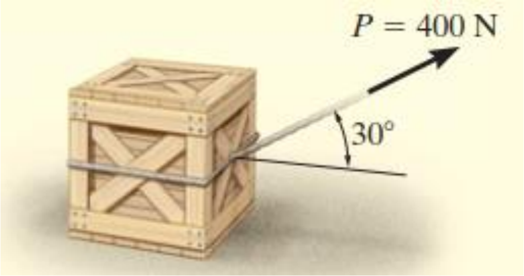
Friction

Friction always acts in the opposite direction of the motion.



Kinetics: Force & Acceleration

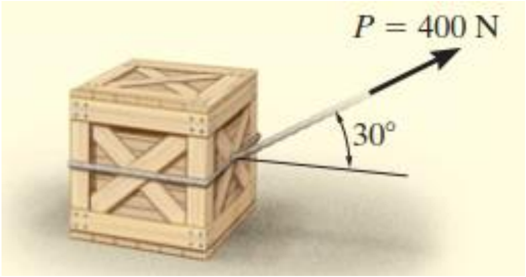
Numerical Example:

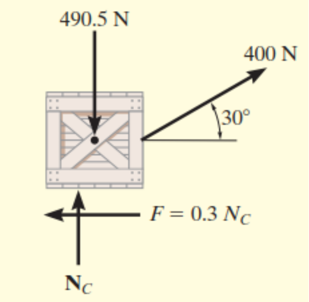


The 50-kg crate shown rests on a horizontal surface for which the coefficient of kinetic friction is μk=0.3 If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

Kinetics: Force & Acceleration

Numerical Example:





Free-Body Diagram. The weight of the crate is $W = mg = 50 \text{ kg } (9.81 \text{ m/s}^2) = 490.5 \text{ N}$. As shown in Fig. 13–6*b*, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration **a** is assumed to act horizontally, in the positive x direction. There are two unknowns, namely N_C and a.

Equations of Motion. Using the data shown on the free-body diagram, we have

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 400 \cos 30^\circ - 0.3N_C = 50a \qquad (1)$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400 \sin 30^\circ = 0 \qquad (2)$$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving for a yields

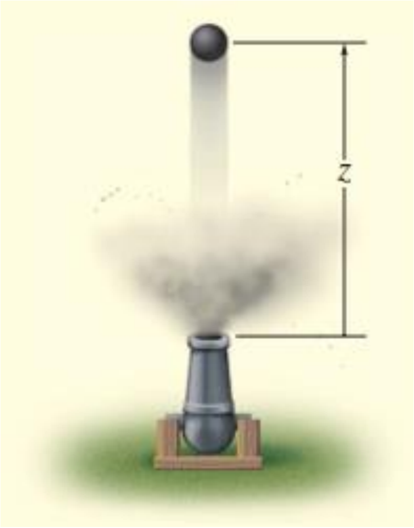
$$N_C = 290.5 \text{ N}$$
$$a = 5.185 \text{ m/s}^2$$

Kinematics. Notice that the acceleration is *constant*, since the applied force **P** is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 5.185(3)$$
$$= 15.6 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

Kinetics: Force & Acceleration

Numerical Example:



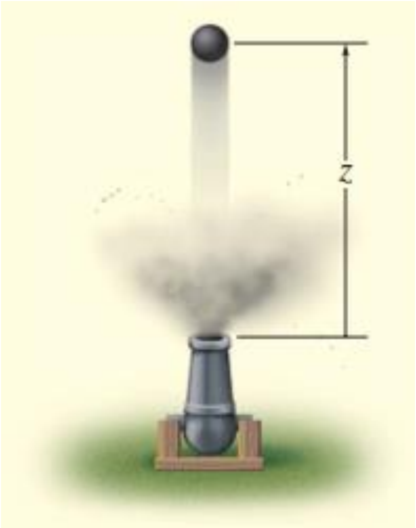
A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s. Determine the maximum height to which it will travel if (a) atmospheric resistance is neglected; and (b) atmospheric resistance is measured as

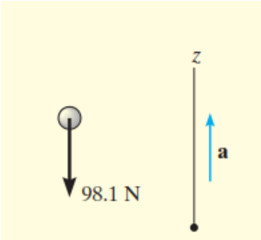
$$F_D = (0.01v^2)\ \mathrm{N}$$

where v is the speed of the projectile at any instant, measured in m/s.

Kinetics: Force & Acceleration

Numerical Example:





No air resistance

SOLUTION

In both cases the known force on the projectile can be related to its acceleration using the equation of motion. Kinematics can then be used to relate the projectile's acceleration to its position.

Part (a) Free-Body Diagram. As shown in Fig. 13–7*b*, the projectile's weight is $W = mg = 10(9.81) = 98.1$ N. We will assume the unknown acceleration **a** acts upward in the *positive z* direction.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \qquad -98.1 = 10a, \qquad a = -9.81 \text{ m/s}^2$$

The result indicates that the projectile, like every object having free-flight motion near the earth's surface, is subjected to a *constant* downward acceleration of 9.81 m/s^2.

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50$ m/s, and at the maximum height $z = h$, $v = 0$. Since the acceleration is *constant*, then

$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(z - z_0)$$

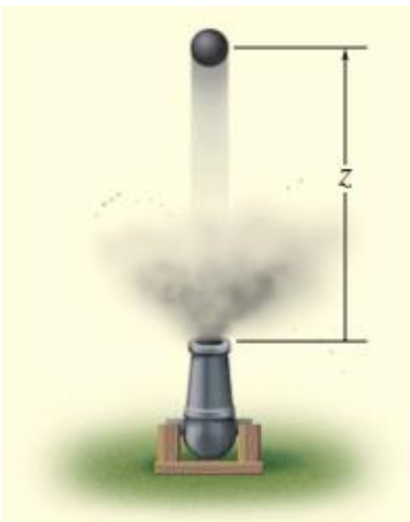
$$0 = (50)^2 + 2(-9.81)(h - 0)$$

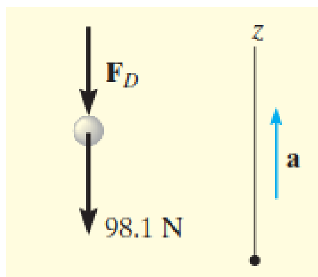
$$h = 127 \text{ m} \qquad \textit{Ans.}$$

Kinetics: Force & Acceleration

Numerical Example:

With air resistance





Part (b) Free-Body Diagram. Since the force $F_D = (0.01v^2)$ N tends to retard the upward motion of the projectile, it acts downward as shown on the free-body diagram, Fig. 13–7*c*.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \qquad -0.01v^2 - 98.1 = 10a, \qquad a = -(0.001v^2 + 9.81)$$

Kinematics. Here the acceleration is *not constant* since F_D depends on the velocity. Since $a = f(v)$, we can relate a to position using

$$(+\uparrow)\ a\,dz = v\,dv; \qquad -(0.001v^2 + 9.81)\,dz = v\,dv$$

Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50$ m/s (positive upward), and at $z = h$, $v = 0$, we have

$$\int_0^h dz = -\int_{50}^0 \frac{v\,dv}{0.001v^2 + 9.81} = -500 \ln(v^2 + 9810)\Big|_{50\text{ m/s}}^{0}$$

$$h = 114 \text{ m} \qquad\qquad \textit{Ans.}$$

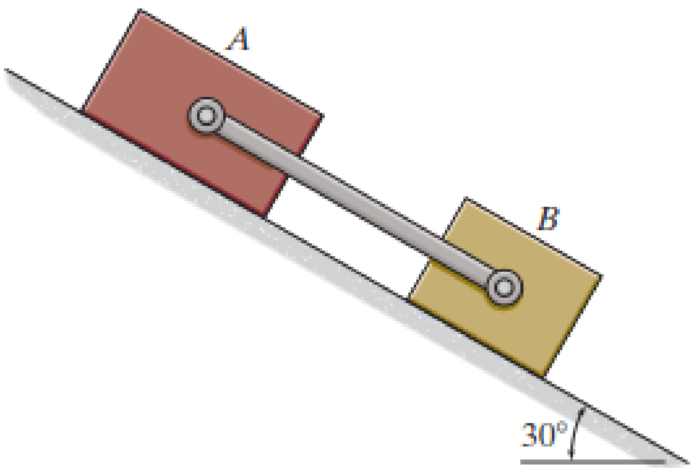
$u = v^2 + 9810$

$du = 2v\,dv$

Variable transformation

Kinetics: Force & Acceleration

Numerical Example:



If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.

Kinetics: Force & Acceleration

Numerical Example:

Free-Body Diagram: Here, the kinetic friction $(F_f)_A = \mu_A N_A = 0.1N_A$ and $(F_f)_B = \mu_B N_B = 0.3N_B$ are required to act up the plane to oppose the motion of the blocks which are down the plane. Since the blocks are connected, they have a common acceleration **a**.

Equations of Motion: By referring to Figs. (a) and (b),

$+\nearrow\Sigma F_{y'} = ma_{y'};\qquad N_A - 10(9.81)\cos 30^\circ = 10(0)$

$$N_A = 84.96\text{ N}$$

$\searrow+\Sigma F_{x'} = ma_{x'};\qquad 10(9.81)\sin 30^\circ - 0.1(84.96) - F = 10a$

$$40.55 - F = 10a$$

and

$+\nearrow\Sigma F_{y'} = ma_{y'};\qquad N_B - 6(9.81)\cos 30^\circ = 6(0)$

$$N_B = 50.97\text{ N}$$

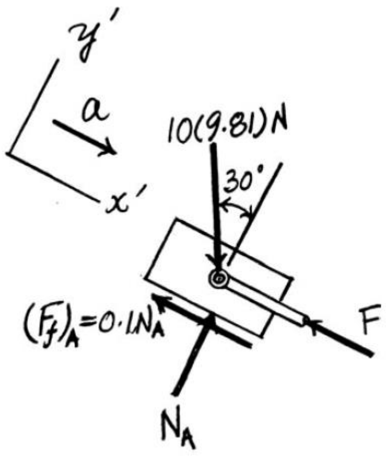
$\searrow+\Sigma F_{x'} = ma_{x'};\qquad F + 6(9.81)\sin 30^\circ - 0.3(50.97) = 6a$

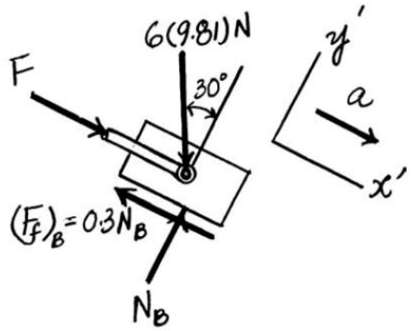
$$F + 14.14 = 6a$$

Solving Eqs. (1) and (2) yields

$$a = 3.42\text{ m/s}^2$$

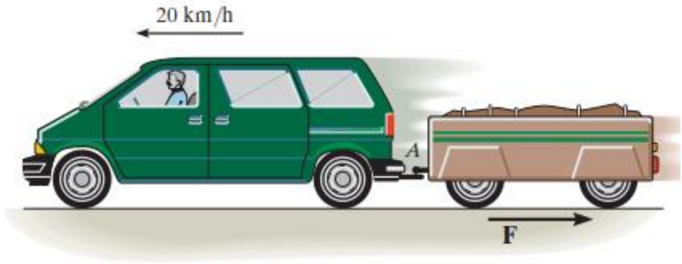
$$F = 6.37\text{ N}$$





Kinetics: Force & Acceleration

Numerical Example:



The van is traveling at 20 km/h when the coupling of the trailer at *A* fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force *F* created by rolling friction which causes the trailer to stop.

Kinetics: Force & Acceleration

Numerical Example:

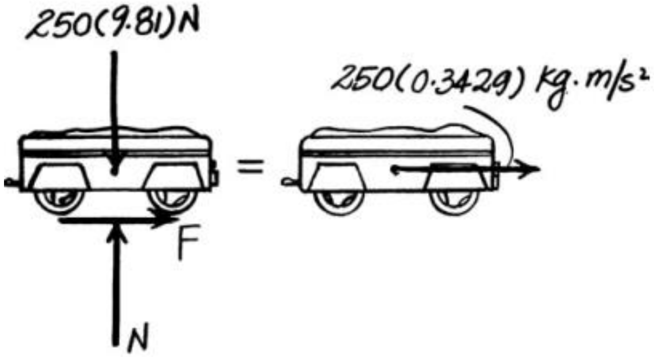
$$20\text{ km/h} = \frac{20(10^3)}{3600} = 5.556\text{ m/s}$$

$$(\overset{+}{\leftarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 5.556^2 + 2(a)(45 - 0)$$

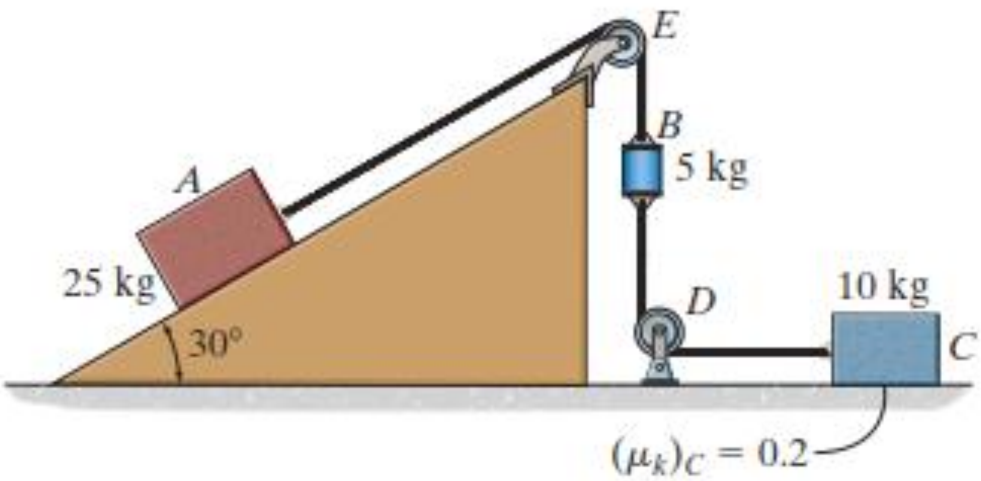
$$a = -0.3429\text{ m/s}^2 = 0.3429\text{ m/s}^2 \rightarrow$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 250(0.3429) = 85.7\text{ N}$$



Kinetics: Force & Acceleration

Numerical Example:

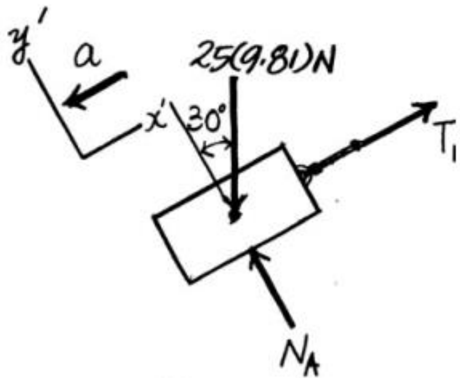


Determine the acceleration of the system and the tension in each cable. The inclined plane is smooth, and the coefficient of kinetic friction between the horizontal surface and block C is $(\mu_k)_C = 0.2$.

Kinetics: Force & Acceleration

Numerical Example:

Free-Body Diagram: The free-body diagram of block A, cylinder B, and block C are shown in Figs. (a), (b), and (c), respectively. The frictional force $(F_f)_C = (\mu_k)_C N_C = 0.2N_C$ must act to the right to oppose the motion of block C which is to the left.



Equations of Motion: Since block A, cylinder B, and block C move together as a single unit, they share a common acceleration **a**. By referring to Figs. (a), (b), and (c),

$$\Sigma F_{x'} = ma_{x'}; \qquad T_1 - 25(9.81)\sin 30^\circ = 25(-a) \qquad \textbf{(1)}$$

and

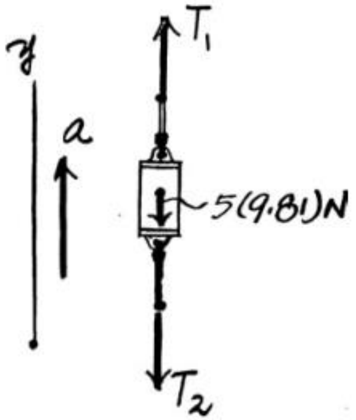
$$+\uparrow \Sigma F_y = ma_y; \qquad T_1 - T_2 - 5(9.81) = 5(a) \qquad \textbf{(2)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 10(9.81) = 10(0)$$

$$N_C = 98.1 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -T_2 + 0.2(98.1) = 10(-a) \qquad \textbf{(3)}$$



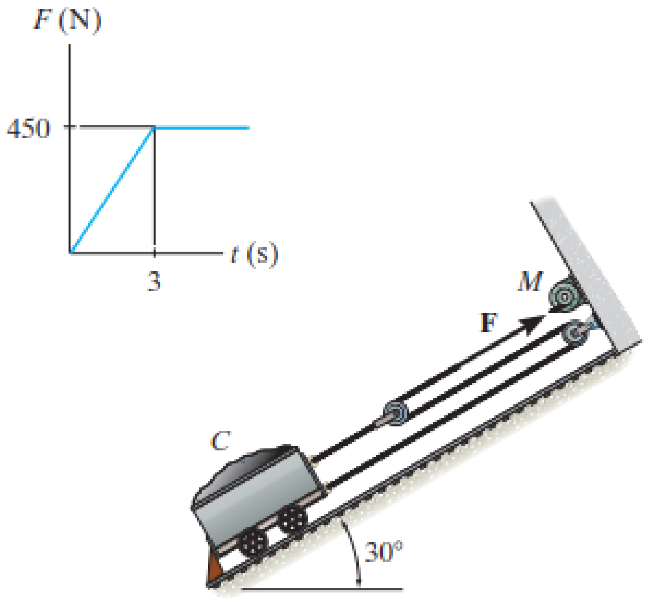
Solving Eqs. (1), (2), and (3), yields

$$a = 1.349 \text{ m/s}^2 \qquad T_1 = 88.90 \text{ N} = 88.9\text{N} \qquad \textbf{Ans.}$$

$$T_2 = 33.11 \text{ N} = 33.1 \text{ N} \qquad \textbf{Ans.}$$

Kinetics: Force & Acceleration

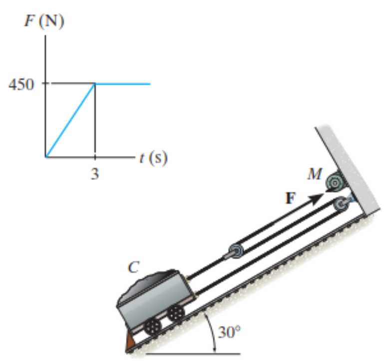
Numerical Example:

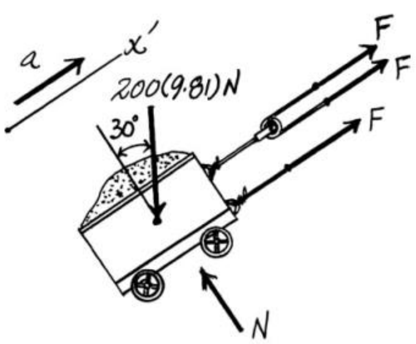


If the force of the motor M on the cable is shown in the graph, determine the velocity of the cart when $t = 3$ s. The load and cart have a mass of 200 kg and the car starts from rest.

Kinetics: Force & Acceleration

Numerical Example:





Equations of Motion: For $0 \le t < 3$ s, $F = \frac{450}{3}t = (150t)$ N. By referring to Fig. we can write

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3(150t) - 200(9.81)\sin 30^\circ = 200a$$

$$a = (2.25t - 4.905)\ \text{m/s}^2$$

For $t > 3$ s, $F = 450$ N. Thus,

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3(450) - 200(9.81)\sin 30^\circ = 200a$$

$$a = 1.845\ \text{m/s}^2$$

Equilibrium: For the rail car to move, force **3F** must overcome the weight component of the rail crate. Thus, the time required to move the rail car is given by

$$\Sigma F_{x'} = 0; \quad 3(150t) - 200(9.81)\sin 30^\circ = 0 \qquad t = 2.18\ \text{s}$$

Kinematics: The velocity of the rail car can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $2.18\ \text{s} \le t < 3$ s, $v = 0$ at $t = 2.18$ s will be used as the integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

$$\int_0^v dv = \int_{2.18\,\text{s}}^{t} (2.25t - 4.905)dt$$

$$v = \left(1.125t^2 - 4.905t\right)\Big|_{2.18\,\text{s}}^{t}$$

$$= \left(1.125t^2 - 4.905t + 5.34645\right)\text{m/s}$$

When $t = 3$ s,

$$v = 1.125(3)^2 - 4.905(3) + 5.34645 = 0.756\text{m/s} \qquad \textbf{Ans.}$$

Course Reference: