Karadeniz Teknik Üniversitesi

Engineering Faculty
Mining Engineering Department

# MINE1000-DYNAMICS

## Week 6:
## Kinetics of a Particle: Impulse and Momentum

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# Course Content

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# Kinetics: Impulse & Momentum

## Principle of Linear Impulse and Momentum

**Momentum:** **tendency of objects to keep going in the same direction with the same speed**

$$\mathbf{L} = m\mathbf{v}$$

**Impulse:** **is the amount of change in an object's momentum**

$$\mathbf{I} = \int \mathbf{F}\, dt$$

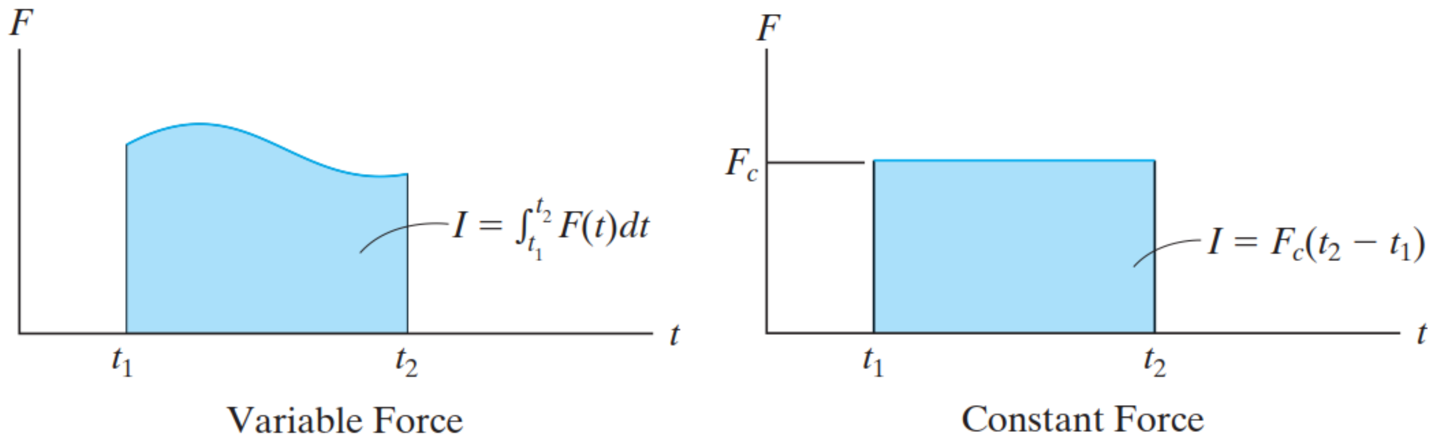
# Kinetics: Impulse & Momentum

Principle of linear impulse and momentum:

$$\Sigma \int_{t_1}^{t_2} \mathbf{F}\,dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

# Kinetics: Impulse & Momentum

$$\mathbf{I} = \int_{t_1}^{t_2} \mathbf{F}_c dt = \mathbf{F}_c(t_2 - t_1).$$



# Kinetics: Impulse & Momentum

## Principle of Linear Impulse and Momentum

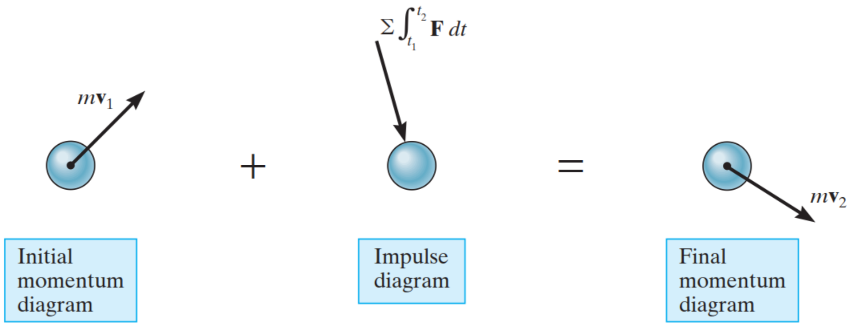
$$m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$$

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$m(v_z)_1 + \Sigma \int_{t_1}^{t_2} F_z\, dt = m(v_z)_2$$

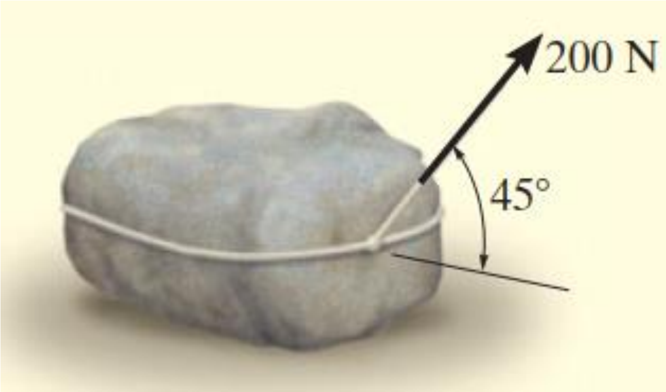
**which states that the initial momentum of the particle at time t1 plus the sum of all the impulses applied to the particle from t1 to t2 is equivalent to the final momentum of the particle at time t2**



# Kinetics: Impulse & Momentum

Numerical Example:

The 100-kg stone shown is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied to the stone for 10 s, determine the final velocity and the normal force which the surface exerts on the stone during this time interval.



# Kinetics: Impulse & Momentum

Solution:

**Principle of Impulse and Momentum.** Applying Eqs. 15–4 yields

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

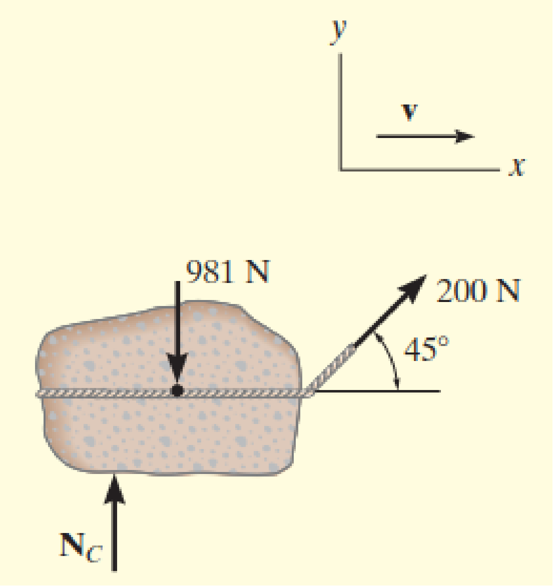
$$0 + 200\text{ N} \cos 45^\circ (10\text{ s}) = (100\text{ kg})v_2$$

$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N} \sin 45^\circ (10\text{ s}) = 0$$

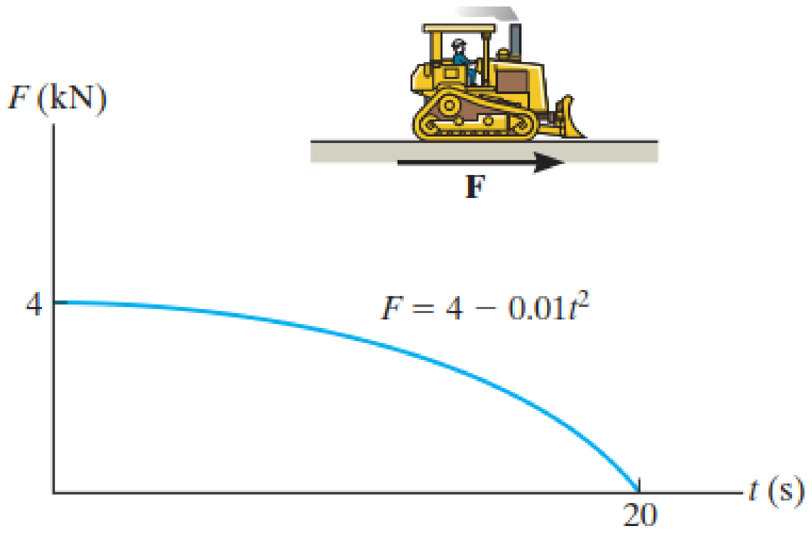
$$N_C = 840\text{ N} \qquad \textit{Ans.}$$



# Kinetics: Impulse & Momentum

Numerical Example:

The 28-Mg bulldozer is originally at rest. Determine its speed when t = 4 s if the horizontal traction F varies with time as shown in the graph.



# Kinetics: Impulse & Momentum

Solution:

The 28-Mg bulldozer is originally at rest. Determine its speed when t = 4 s if the horizontal traction F varies with time as shown in the graph.

$$m(v_x)_1 + \Sigma \int_{t_1}^{-t_2} F_x \, dt = m(v_x)_2$$

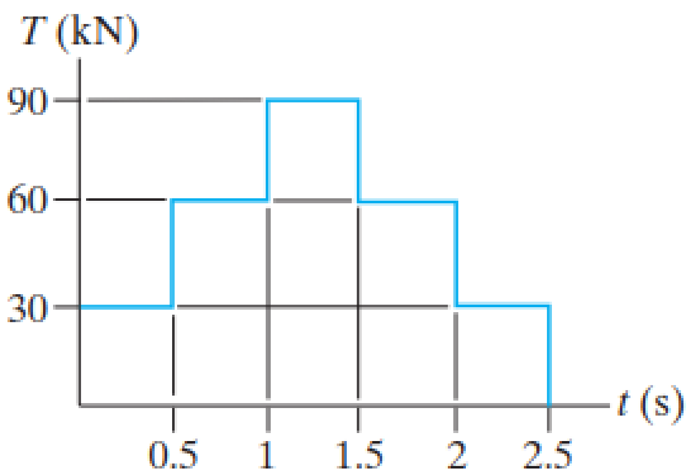
$$0 + \int_0^4 (4 - 0.01t^2)(10^3)dt = 28(10^3)v$$

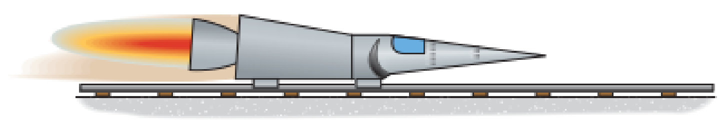
$$v = 0.564 \text{ m/s}$$

# Kinetics: Impulse & Momentum

Numerical Example:

Determine the maximum speed attained by the 1.5-Mg rocket sled if the rockets provide the thrust shown in the graph. Initially, the sled is at rest. Neglect friction and the loss of mass due to fuel consumption.



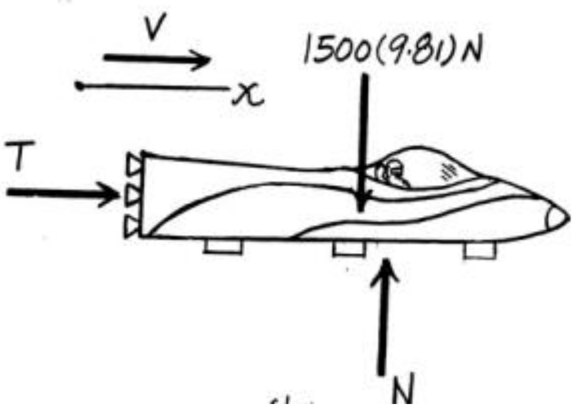



# Kinetics: Impulse & Momentum

Solution:

**Principle of Impulse and Momentum:** The graph of thrust **T** vs. time $t$ due to the successive ignition of the rocket is shown in Fig. $a$. The sled attains its maximum speed at the instant that all the rockets burn out their fuel, that is, at $t = 2.5$ s. The impulse generated by **T** during $0 \leq t \leq 2.5$ s is equal to the area under the $T$ vs $t$ graphs. Thus,

$$I = \int T\,dt = 30(10^3)(0.5 - 0) + 60(10^3)(1 - 0.5) + 90(10^3)(1.5 - 1)$$
$$+ 60(10^3)(2 - 1.5) + 30(10^3)(25 - 2) = 135\,000\ \text{N}\cdot\text{s}$$




By referring to the free-body diagram of the sled shown in Fig. $a$,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int F_x\,dt = m(v_2)_x$$

$$1500(0) + 135000 = 1500 v_{max}$$

$$v_{max} = 90\ \text{m/s} \qquad \textbf{Ans.}$$

# Kinetics: Impulse & Momentum

Conservation of Linear Momentum:

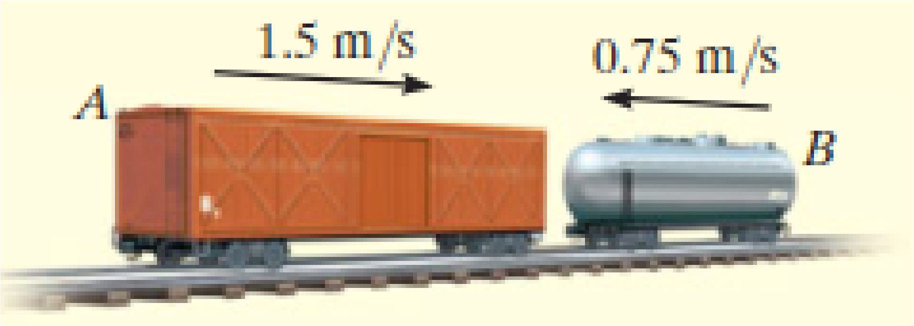
When the sum of the external impulses acting on a system of particles is zero

$$\Sigma m_i(\mathbf{v}_i)_1 = \Sigma m_i(\mathbf{v}_i)_2$$

# Kinetics: Impulse & Momentum

Numerical Example:

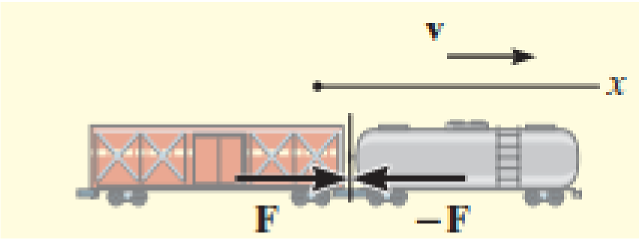
The 15-Mg boxcar A is coasting at on the horizontal track when it encounters a 12-Mg tank car B coasting at toward it as shown. If the cars collide and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in 0.8 s.



# Kinetics: Impulse & Momentum

Solution:

Here we have considered both cars as a single system. By inspection, momentum is conserved in the x direction since the coupling force F is internal to the system and will therefore cancel out. It is assumed both cars, when coupled, move at in the positive x direction.

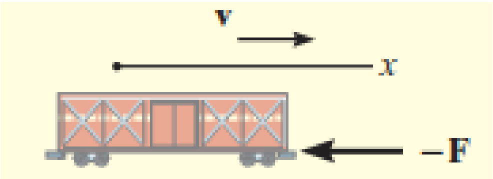



**Conservation of Linear Momentum.**

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$(15\,000\text{ kg})(1.5\text{ m/s}) - 12\,000\text{ kg}(0.75\text{ m/s}) = (27\,000\text{ kg})v_2$$

$$v_2 = 0.5\text{ m/s} \rightarrow \qquad \textit{Ans.}$$




**Principle of Impulse and Momentum.** Since $\int F\,dt = F_{avg}\,\Delta t = F_{avg}(0.8\text{ s})$, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + \Sigma \int F\,dt = m_A v_2$$

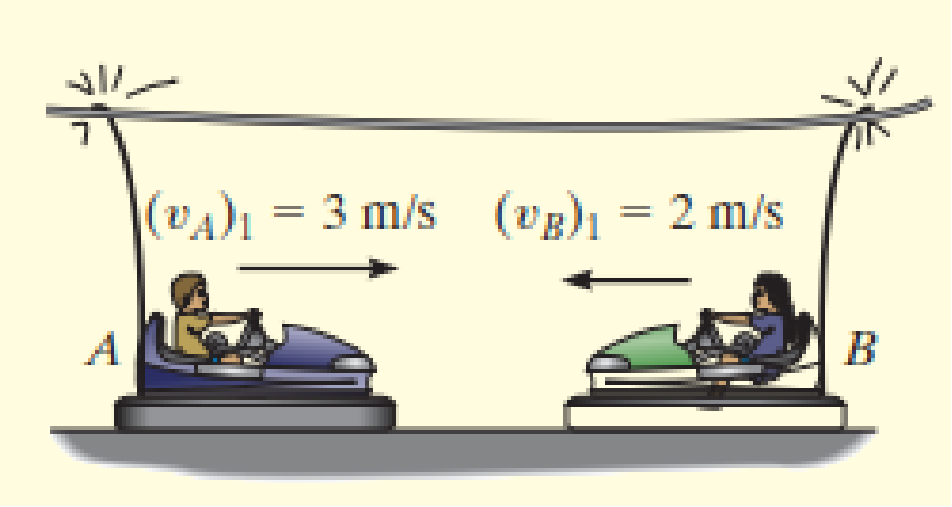
$$(15\,000\text{ kg})(1.5\text{ m/s}) - F_{avg}(0.8\text{ s}) = (15\,000\text{ kg})(0.5\text{ m/s})$$

$$F_{avg} = 18.8\text{ kN} \qquad \textit{Ans.}$$

# Kinetics: Impulse & Momentum

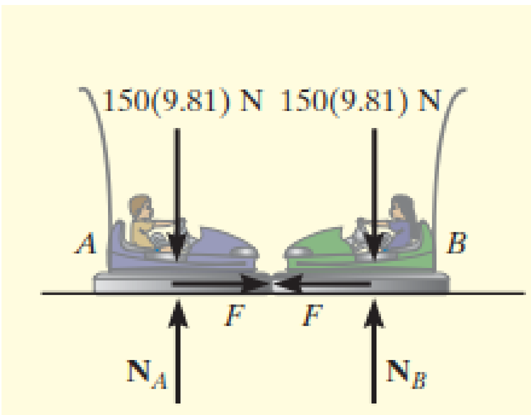
Numerical Example:

The bumper cars A and B each have a mass of 150 kg and are coasting with the velocities shown before they freely collide head on. If no energy is lost during the collision, determine their velocities after collision.



# Kinetics: Impulse & Momentum

Solution:




**Conservation of Momentum.**

$$(\xrightarrow{+})\qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(150\text{ kg})(3\text{ m/s}) + (150\text{ kg})(-2\text{ m/s}) = (150\text{ kg})(v_A)_2 + (150\text{ kg})(v_B)_2$$

$$(v_A)_2 = 1 - (v_B)_2 \qquad (1)$$

**Conservation of Energy.** Since no energy is lost, the conservation of energy theorem gives

$$T_1 + V_1 = T_2 + V_2$$

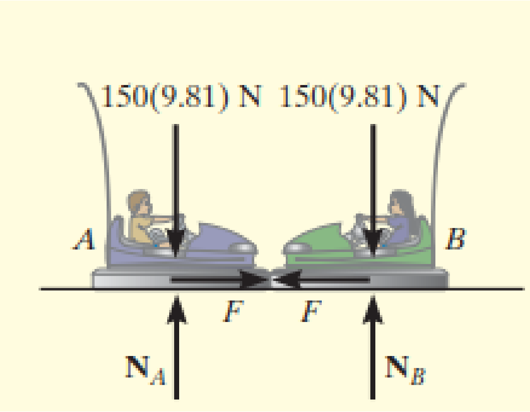
$$\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2 + 0 = \frac{1}{2}m_A(v_A)_2^2 + \frac{1}{2}m_B(v_B)_2^2 + 0$$

$$\frac{1}{2}(150\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(150\text{ kg})(2\text{ m/s})^2 + 0 = \frac{1}{2}(150\text{ kg})(v_A)_2^2 + \frac{1}{2}(150\text{ kg})(v_B)_2{}^2 + 0$$

$$(v_A)_2^2 + (v_B)_2^2 = 13 \qquad (2)$$

# Kinetics: Impulse & Momentum

Solution:




Substituting Eq. (1) into (2) and simplifying, we get

$$(v_B)_2^2 - (v_B)_2 - 6 = 0$$

Solving for the two roots,

$$(v_B)_2 = 3 \text{ m/s} \qquad \text{and} \qquad (v_B)_2 = -2 \text{ m/s}$$

Since $(v_B)_2 = -2$ m/s refers to the velocity of $B$ just *before* collision, then the velocity of $B$ just after the collision must be

$$(v_B)_2 = 3 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

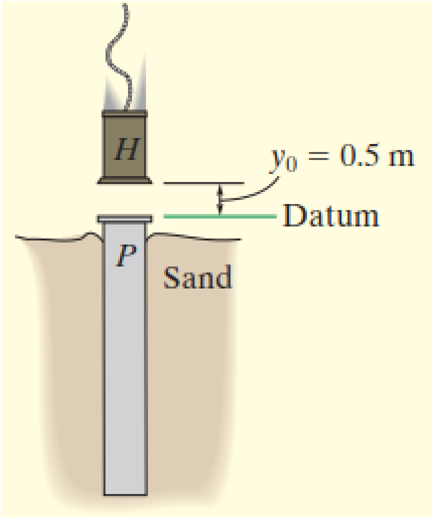
Substituting this result into Eq. (1), we obtain

$$(v_A)_2 = 1 - 3 \text{ m/s} = -2 \text{ m/s} = 2 \text{ m/s} \leftarrow \qquad \textit{Ans.}$$

# Kinetics: Impulse & Momentum

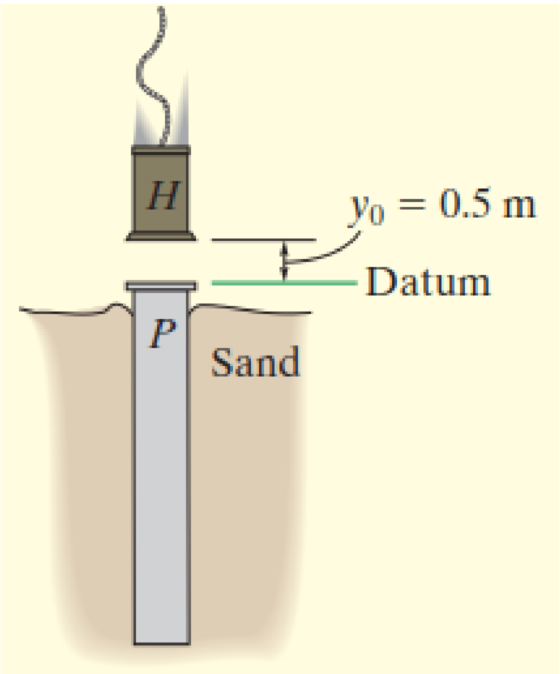
Numerical Example:

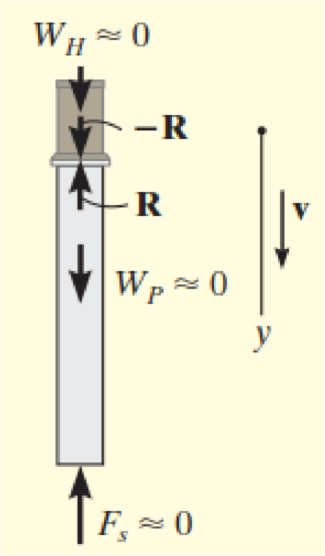
An 800-kg rigid pile shown is driven into the ground using a 300-kg hammer.The hammer falls from rest at a height and strikes the top of the pile. Determine the impulse which the pile exerts on the hammer if the pile is surrounded entirely by loose sand so that after striking, the hammer does not rebound off the pile.

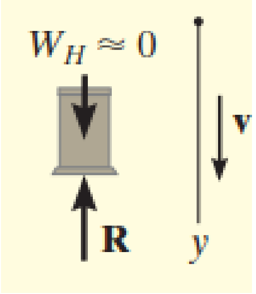


# Kinetics: Impulse & Momentum

Solution:




**Conservation of Momentum.** Since the hammer does not rebound off the pile just after collision, then $(v_H)_2 = (v_P)_2 = v_2$.

$$(+\downarrow) \qquad m_H(v_H)_1 + m_P(v_P)_1 = m_H v_2 + m_P v_2$$

$$(300\text{ kg})(3.132\text{ m/s}) + 0 = (300\text{ kg})v_2 + (800\text{ kg})v_2$$

$$v_2 = 0.8542\text{ m/s}$$

**Principle of Impulse and Momentum.** The impulse which the pile imparts to the hammer can now be determined since $\mathbf{v}_2$ is known. From the free-body diagram for the hammer, Fig. 15–11*c*, we have

$$(+\downarrow) \qquad m_H(v_H)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m_H v_2$$

$$(300\text{ kg})(3.132\text{ m/s}) - \int R\,dt = (300\text{ kg})(0.8542\text{ m/s})$$

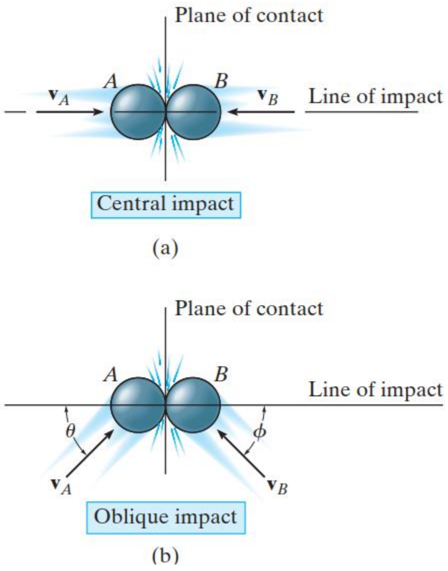
$$\int R\,dt = 683\text{ N}\cdot\text{s} \qquad \textit{Ans.}$$

# Kinetics: Impulse & Momentum

## Impact

Impact occurs when two bodies collide with each other during a very short period of time, causing relatively large (impulsive) forces to be exerted between the bodies. The striking of a hammer on a nail, or a golf club on a ball, are common examples of impact loadings.

$$(\xrightarrow{+}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$




The ratio of the restitution impulse to the deformation impulse is called the coefficient of restitution, *e*.

$$\boxed{e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}}$$

# Kinetics: Impulse & Momentum

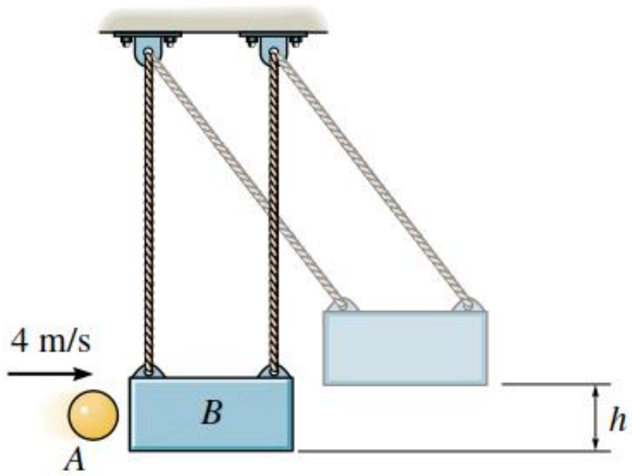
**Elastic Impact ($e = 1$).** If the collision between the two particles is *perfectly elastic*, the deformation impulse $\left(\int \mathbf{P}\,dt\right)$ is equal and opposite to the restitution impulse $\left(\int \mathbf{R}\,dt\right)$. Although in reality this can never be achieved, $e = 1$ for an elastic collision.

**Plastic Impact ($e = 0$).** The impact is said to be *inelastic or plastic* when $e = 0$. In this case there is no restitution impulse $\left(\int \mathbf{R}\,dt = \mathbf{0}\right)$, so that after collision both particles couple or stick *together* and move with a common velocity.

# Kinetics: Impulse & Momentum

Numerical Example:

The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e$ = 0.8, determine the maximum height h to which the block will swing before it momentarily stops.



# Kinetics: Impulse & Momentum

Solution:

$$\left(\xrightarrow{+}\right) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$$\left(\xrightarrow{+}\right) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving:

$(v_A)_2 = -2.545 \text{ m/s}$

$(v_B)_2 = 0.6545 \text{ m/s}$

Block:

Datum at lowest point.

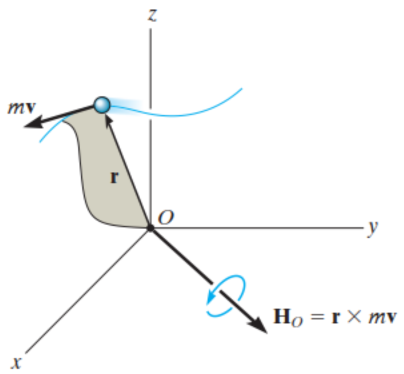
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$$

$$h = 0.0218 \text{ m} = 21.8 \text{ mm}$$

# Kinetics: Impulse & Momentum

## Angular Momentum and Impulse:




### Angular Momentum

$$\boxed{\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}}$$

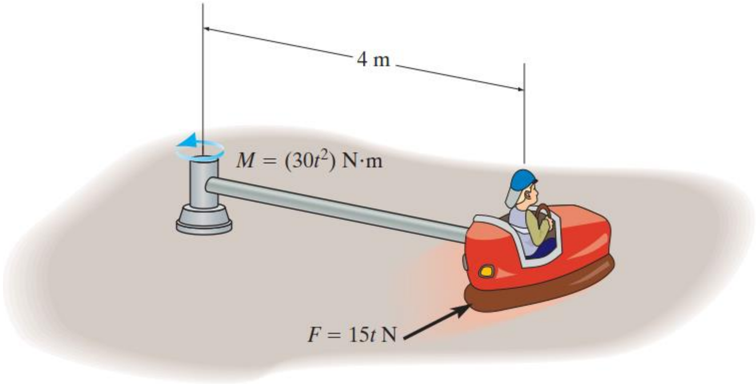
$$\boxed{\text{angular impulse} = \int_{t_1}^{t_2} \mathbf{M}_O\, dt = \int_{t_1}^{t_2} (\mathbf{r} \times \mathbf{F})\, dt}$$

$$(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O\, dt = (\mathbf{H}_O)_2$$

# Kinetics: Impulse & Momentum

## Numerical Example

If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)$ N·m and the engine of the car supplies a traction force of $F = (15t)$ N to the wheels, where $t$ is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



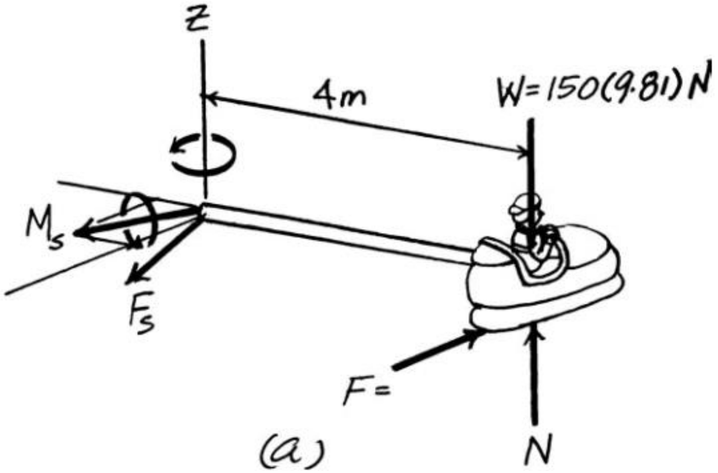
# Kinetics: Impulse & Momentum

Solution:

$$\left(H_1\right)_z + \Sigma \int_{t_2}^{t_1} M_z \, dt = \left(H_2\right)_z$$

$$0 + \int_0^{5\,\mathrm{s}} 30t^2 \, dt + \int_0^{5\,\mathrm{s}} 15t(4)dt = 150v(4)$$

$$v = 3.33 \text{ m/s}$$



**Course Reference:**