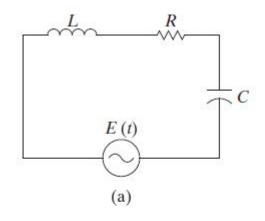
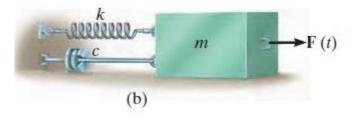
Mechanical Vibrations

Electric Circuit Analogs & Vibration Measurement

Electrical Circuit Analogs

- The characteristics of a vibrating mechanical system can be represented by an electric circuit. Consider the circuit shown in Fig. *a*, which consists of an inductor *L*, a resistor *R*, and a capacitor *C*.
- When a voltage E(t) is applied, it causes a current of magnitude *i* to flow through the circuit. As the current flows past the inductor the voltage drop is L(di/dt), when it flows across the resistor the drop is Ri, and when it arrives at the capacitor the drop is $(1/C) \int i dt$.
- Since current cannot flow past a capacitor, it is only possible to measure the charge q acting on the capacitor. The charge can, however, be related to the current by the equation i = dq/dt. Thus, the voltage drops which occur across the inductor, resistor, and capacitor become L d²q/dt², R dq/dt, and q/C, respectively.





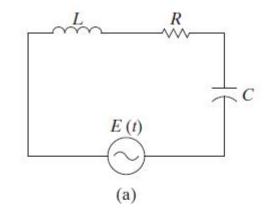
• According to Kirchhoff's voltage law, the applied voltage balances the sum of the voltage drops around the circuit. Therefore,

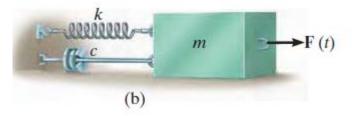
$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$$

- Consider now the model of a single-degree-of-freedom mechanical system, Fig. *b*, which is subjected to both a general forcing function *F*(*t*) and damping.
- The equation of motion for this system was established in the previous section and can be written as

$$n\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

• By comparison, it is seen that the above equations have the same form, and hence mathematically the procedure of analyzing an electric circuit is the same as that of analyzing a vibrating mechanical system. The analogs between the two equations are given in the following table.





Electrical–Mechanical Analogs			
Electrical		Mechanical	
Electric charge	q	Displacement	x
Electric current	i	Velocity	dx/dt
Voltage	E(t)	Applied force	F(t)
Inductance	L	Mass	т
Resistance	R	Viscous damping coefficient	С
Reciprocal of capacitance	1/C	Spring stiffness	k

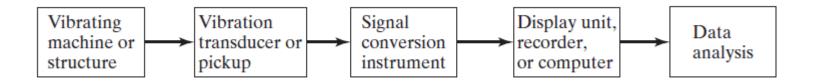
• This analogy has important application to experimental work, for it is much easier to simulate the vibration of a complex mechanical system using an electric circuit, which can be constructed on an analog computer, than to make an equivalent mechanical spring-and-dashpot model.

Vibration Measurement

- In some practical situations, it might be difficult to develop a mathematical model of the system and predict its vibration characteristics through an analytical study.
- In such cases, we can use experimental methods to measure the vibration response of the system to a known input. This helps in identifying the system in terms of its mass, stiffness, and damping.
- In practice, the measurement of vibration becomes necessary for the following reasons:
- 1. <u>The increasing demands of higher productivity and economical design lead to higher operating</u> <u>speeds of machinery and efficient use of materials through lightweight structures.</u> These trends make the occurrence of resonant conditions more frequent during the operation of machinery and reduce the reliability of the system. Hence the periodic measurement of vibration characteristics of machinery and structures becomes essential to ensure adequate safety margins. Any observed shift in the natural frequencies or other vibration characteristics will indicate either a failure or a need for maintenance of the machine.

- 2. <u>The measurement of the natural frequencies of a structure or machine is useful in selecting the operational speeds of nearby machinery to avoid resonant conditions.</u>
- 3. The theoretically computed vibration characteristics of a machine or structure may be different from the actual values due to the assumptions made in the analysis.
- 4. The measurement of frequencies of vibration and the forces developed is necessary in the design and operation of active vibration-isolation systems.
- 5. In many applications, the survivability of a structure or machine in a specified vibration environment is to be determined. If the structure or machine can perform the expected task even after completion of testing under the specified vibration environment, it is expected to survive the specified conditions.
- 6. Continuous systems are often approximated as multi-degree-of-freedom systems for simplicity. If the measured natural frequencies and mode shapes of a continuous system are comparable to the computed natural frequencies and mode shapes of the multi-degree-of-freedom model, then the approximation will be proved to be a valid one.
- 7. The measurement of input and the resulting output vibration characteristics of a system helps in identifying the system in terms of its mass, stiffness, and damping.
- 8. The information about ground vibrations due to earthquakes, fluctuating wind velocities on structures, random variation of ocean waves, and road surface roughness are important in the design of structures, machines, oil platforms, and vehicle suspension systems.

• Vibration measurement scheme. The figure shown illustrates the basic features of a vibration measurement scheme.

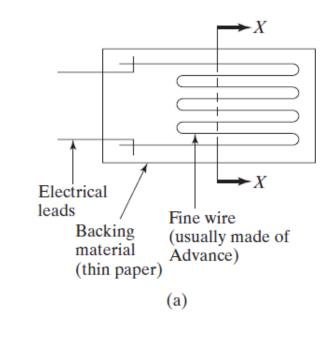


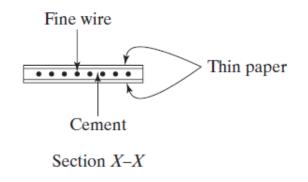
- In this figure, the motion (or dynamic force) of the vibrating body is converted into an electrical signal by the vibration transducer or pickup. In general, a transducer is a device that transforms changes in mechanical quantities (such as displacement, velocity, acceleration, or force) into changes in electrical quantities (such as voltage or current).
- Since the output signal (voltage or current) of a transducer is too small to be recorded directly, a signal conversion instrument is used to amplify the signal to the required value.
- The output from the signal conversion instrument can be presented on a display unit for visual inspection, or recorded by a recording unit, or stored in a computer for later use.
- The data can then be analyzed to determine the desired vibration characteristics of the machine or structure.

- Depending on the quantity measured, a vibration measuring instrument is called a vibrometer, a velocity meter, an accelerometer, a phase meter, or a frequency meter.
- If the instrument is designed to record the measured quantity, then the suffix "meter" is to be replaced by "graph".
- In some application, we need to vibrate a machine or structure to find its resonance characteristics. For this, electrodynamic vibrators, electrohydraulic vibrators, and signal generators (oscillators) are used.
- The following considerations often dictate the type of vibration-measuring instruments to be used in a vibration test:
- 1. expected ranges of the frequencies and amplitudes,
- 2. sizes of the machine/structure involved,
- 3. conditions of operation of the machine/equipment/structure, and
- 4. type of data processing used (such as graphical display or graphical recording or storing the record in digital form for computer processing).

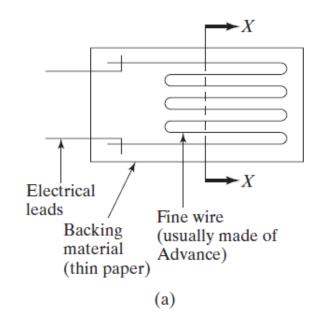
Transducers

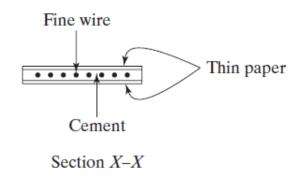
- A transducer is a device that transforms values of physical variables into equivalent electrical signals. Several types of transducers are available; some of them are less useful than others due to their nonlinearity or slow response.
- Some of the transducers commonly used for vibration measurement are discussed below.
- Variable resistance transducers. In these transducers, a mechanical motion produces a change in electrical resistance (of a rheostat, a strain gage, or a semiconductor), which in turn causes a change in the output voltage or current.
- The schematic diagram of an electrical resistance strain gage is shown.



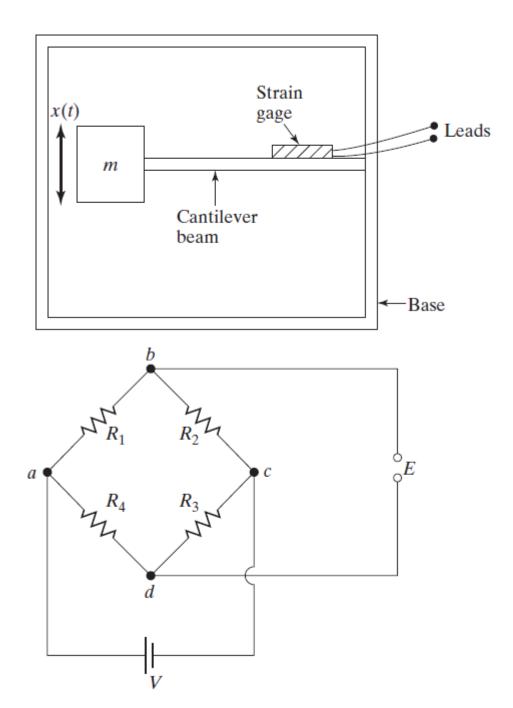


- An electrical resistance strain gage consists of a fine wire whose resistance changes when it is subjected to mechanical deformation.
- When the strain gage is bonded to a structure, it experiences the same motion (strain) as the structure and hence its resistance change gives the strain applied to the structure.
- The wire is sandwiched between two sheets of thin paper. The strain gage is bonded to the surface where the strain is to be measured.
- The most common gage material is a copper-nickel alloy known as Advance.

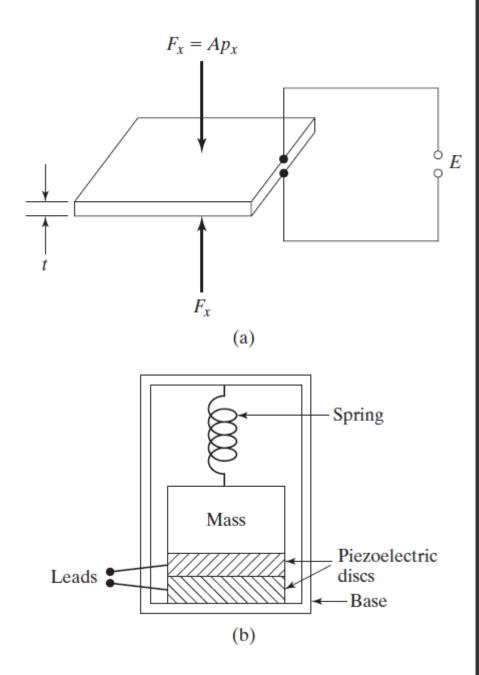




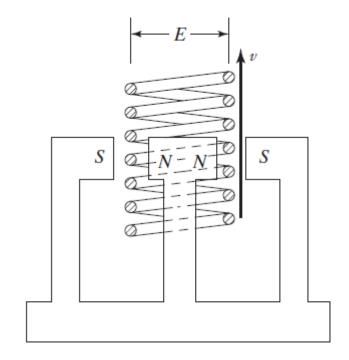
- In a vibration pickup, the strain gage is mounted on an elastic element of a spring-mass system as shown.
- The strain at any point on the cantilever (elastic member) is proportional to the deflection of the mass, *x*(*t*), to be measured. Hence the strain indicated by the strain gage can be used to find *x*(*t*).
- The change in resistance of the wire *R* can be measured using a Wheatstone bridge, potentiometer circuit, or voltage divider.
- A typical Wheatstone bridge, representing a circuit which is sensitive to small changes in the resistance, is shown. A d.c. voltage *V* is applied across the points *a* and *c*.



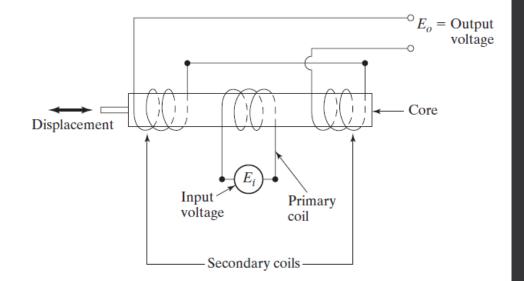
- **Piezoelectric transducers.** Certain natural and manufactured materials like quartz, tourmaline, lithium sulfate, and Rochelle salt generate electrical charge when subjected to a deformation or mechanical stress (Fig. a). The electrical charge disappears when the mechanical loading is removed.
- Such materials are called piezoelectric materials and the transducers, which take advantage of the piezoelectric effect, are known as piezoelectric transducers.
- A typical piezoelectric transducer (accelerometer) is shown in Fig. b. In this figure, a small mass is spring loaded against a piezoelectric crystal. When the base vibrates, the load exerted by the mass on the crystal changes with acceleration, hence the output voltage generated by the crystal will be proportional to the acceleration.
- The main advantages of the piezoelectric accelerometer include compactness, ruggedness, high sensitivity, and high frequency range.



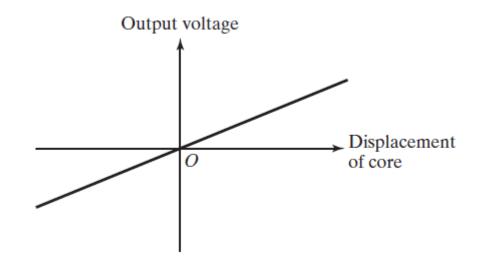
- Electrodynamic transducers. When an electrical conductor, in the form of a coil, moves in a magnetic field as shown, a voltage *E* is generated in the conductor.
- The magnetic field may be produced by either a permanent magnet or an electromagnet. Sometimes, the coil is kept stationary and the magnet is made to move.
- Since the voltage output of an electromagnetic transducer is proportional to the relative velocity of the coil, they are frequently used in "velocity pickups."



- Linear variable differential transformer transducer. The schematic diagram of a *linear* variable differential transformer (LVDT) transducer is shown. It consists of a primary coil at the center, two secondary coils at the ends, and a magnetic core that can move freely inside the coils in the axial direction.
- When an a.c. input voltage is applied to the primary coil, the output voltage will be equal to the difference of the voltages induced in the secondary coils. This output voltage depends on the magnetic coupling between the coils and the core, which in turn depends on the axial displacement of the core. The secondary coils are connected in phase opposition so that, when the magnetic core is in the exact middle position, the voltages in the two coils will be equal and 180° out of phase. This makes the output voltage of the LVDT as zero. When the core is moved to either side of the middle (zero) position, the magnetic coupling will be increased in one secondary coil and decreased in the other coil. The output polarity depends on the direction of the movement of the magnetic core.

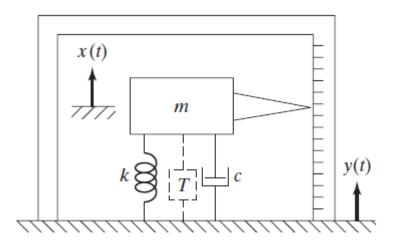


- The range of displacement for many LVDTs on the market is from 0.0002 cm to 40 cm. The advantages of an LVDT over other displacement transducers include insensitivity to temperature and high output. The mass of the magnetic core restricts the use of the LVDT for high-frequency applications.
- As long as the core is not moved very far from the center of the coil, the output voltage varies linearly with the displacement of the core as shown; hence the name linear variable differential transformer.

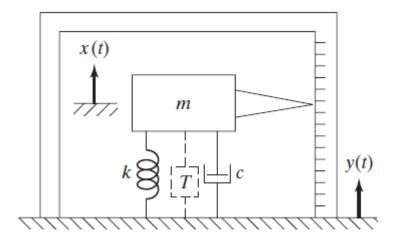


Vibration Pickups

- When a transducer is used in conjunction with another device to measure vibrations, it is called a *vibration pickup*.
- The commonly used vibration pickups are known as seismic instruments. A seismic instrument consists of a mass-spring-damper system mounted on the vibrating body, as shown. Then the vibratory motion is measured by finding the displacement of the mass relative to the base on which it is mounted.



- The instrument consists of a mass *m*, a spring *k*, and a damper *c* inside a cage, which is fastened to the vibrating body.
- With this arrangement, the bottom ends of the spring and the dashpot will have the same motion as the cage (which is to be measured, *y*) and their vibration excites the suspended mass into motion. Then the displacement of the mass relative to the cage, *z* = *x y*, where *x* denotes the vertical displacement of the suspended mass, can be measured if we attach a pointer to the mass and a scale to the cage, as shown.



• The vibrating body is assumed to have a harmonic motion:

$$y(t) = Y \sin \omega t \tag{a}$$

• The equation of motion of the mass *m* can be written as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$
 (b)

• By defining the relative displacement *z* as

$$z = x - y \tag{c}$$

• Eq. (b) can be written as

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{d}$$

• Equations (a) and (d) lead to

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 Y \sin \omega t$$
 (e)

• The steady-state solution is given by

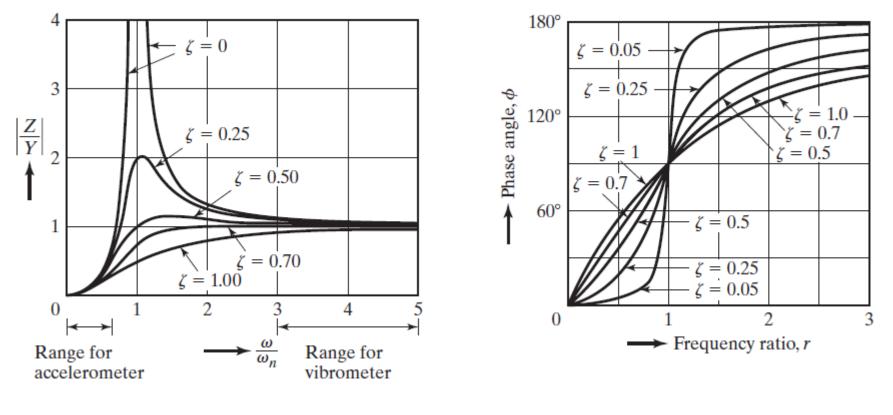
$$z(t) = Z \sin(\omega t - \phi) \tag{f}$$

• where Z and ϕ are given by

$$Z = \frac{Y\omega^2}{[(k - m\omega^2)^2 + c^2\omega^2]^{1/2}} = \frac{r^2Y}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}}$$
$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right) = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right)$$
$$r = \frac{\omega}{\omega_n}$$

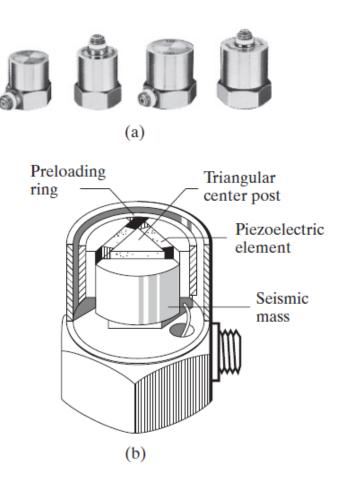
$$\zeta = \frac{c}{2m\omega_n}$$

• The variations of Z and ϕ with respect to r are shown in Figs. (a) and (b). As will be seen later, the type of instrument is determined by the useful range of the frequencies, indicated in Fig. (a).



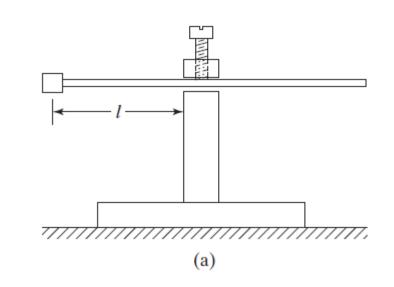
(a)

- **Vibrometer.** A *vibrometer* or a *seismometer* is an instrument that measures the displacement of a vibrating body.
- Accelerometer. An accelerometer is an instrument that measures the acceleration of a vibrating body.
- Accelerometers are widely used for vibration measurements and also to record earthquakes. From the accelerometer record, the velocity and displacements are obtained by integration.
- **Velometer.** A velometer measures the velocity of a vibrating body.
- **Phase distortion.** All vibration-measuring instruments exhibit phase lag. Thus the response or output of the instrument lags behind the motion or input it measures. The time lag is given by the phase angle divided by the frequency ω . The time lag is not important if we measure a single harmonic component. But, occasionally, the vibration to be recorded is not harmonic but consists of the sum of two or more harmonic components. In such a case, the recorded graph may not give an accurate picture of the vibration, because different harmonics may be amplified by different amounts and their phase shifts may also be different.
- The distortion in the waveform of the recorded signal is called the *phase distortion* or *phase-shift error*.

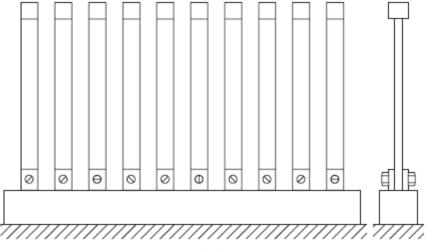


Frequency-Measuring Instruments

- Most frequency-measuring instruments are of the mechanical type and are based on the principle of resonance. Two kinds are discussed in the following paragraphs: the Fullarton tachometer and the Frahm tachometer.
- Single-Reed Instrument or Fullarton Tachometer. This instrument consists of a variable-length cantilever strip with a mass attached at one of its ends. The other end of the strip is clamped, and its free length can be changed by means of a screw mechanism (see Fig. a). Since each length of the strip corresponds to a different natural frequency, the reed is marked along its length in terms of its natural frequency.
- In practice, the clamped end of the strip is pressed against the vibrating body, and the screw mechanism is manipulated to alter its free length until the free end shows the largest amplitude of vibration. At that instant, the excitation frequency is equal to the natural frequency of the cantilever; it can be read directly from the strip.



- **Multireed-Instrument or Frahm Tachometer.** This instrument consists of a number of cantilevered reeds carrying small masses at their free ends (see Fig. b). Each reed has a different natural frequency and is marked accordingly. Using a number of reeds makes it possible to cover a wide frequency range.
- When the instrument is mounted on a vibrating body, the reed whose natural frequency is nearest the unknown frequency of the body vibrates with the largest amplitude. The frequency of the vibrating body can be found from the known frequency of the vibrating reed.



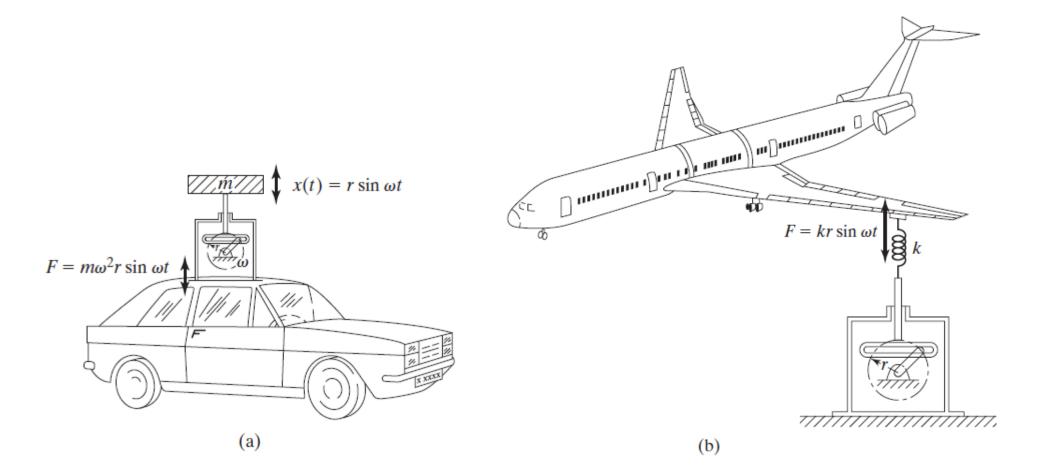
- **Stroboscope.** A stroboscope is an instrument that produces light pulses intermittently. The frequency at which the light pulses are produced can be altered and read from the instrument.
- When a specific point on a rotating (vibrating) object is viewed with the stroboscope, it will appear to be stationary only when the frequency of the pulsating light is equal to the speed of the rotating (vibrating) object.
- The main advantage of the stroboscope is that it does not make contact with the rotating (vibrating) body. Due to the persistence of vision, the lowest frequency that can be measured with a stroboscope is approximately 15 Hz.



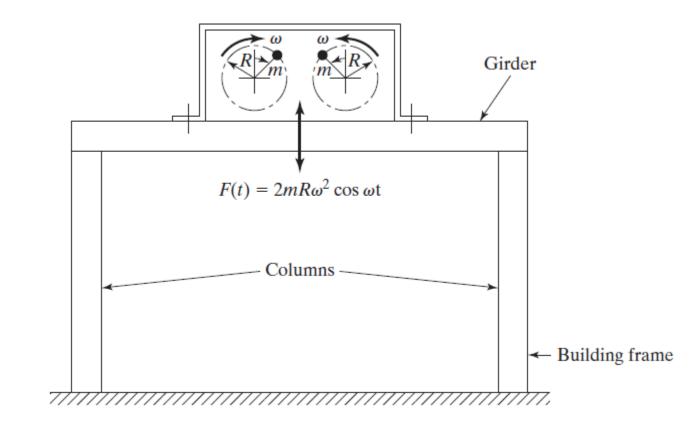
Vibration Exciters

- The vibration exciters or shakers can be used in several applications such as determination of the dynamic characteristics of machines and structures and fatigue testing of materials.
- The vibration exciters can be mechanical, electromagnetic, electrodynamic, or hydraulic type.
- **Mechanical exciters.** A Scotch yoke mechanism can be used to produce harmonic vibrations. The crank of the mechanism can be driven either by a constant- or a variable-speed motor.

• When a structure is to be vibrated, the harmonic force can be applied either as an inertia force, as shown in Fig. (a), or as an elastic spring force, as shown in Fig. (b). These vibrators are generally used for frequencies less than 30 Hz and loads less than 700 N.



• The unbalance created by two masses rotating at the same speed in opposite directions can be used as a mechanical exciter. This type of shaker can be used to generate relatively large loads between 250 N and 25,000 N.



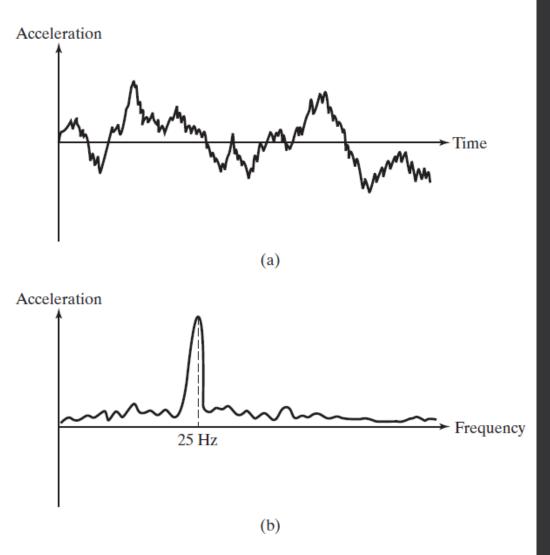
- Electrodynamic shaker. The electrodynamic shaker can be considered as the reverse of an electrodynamic transducer.
- The electrodynamic exciters are used to generate forces up to 30,000 N, displacements up to 25 mm, and frequencies in the range of 5 Hz to 20 kHz.



Signal Analysis

- In signal analysis, we determine the response of a system under a known excitation and present it in a convenient form. Often, the time response of a system will not give much useful information. However, the frequency response will show one or more discrete frequencies around which the energy is concentrated.
- Since the dynamic characteristics of individual components of the system are usually known, we can relate the distinct frequency components (of the frequency response) to specific components.

- For example, the acceleration-time history of a machine frame that is subjected to excessive vibration might appear as shown in Fig. (a). This figure cannot be used to identify the cause of vibration.
- If the acceleration-time history is transformed to the frequency domain, the resulting frequency spectrum might appear as shown in Fig. (b), where, for specificness, the energy is shown concentrated around 25 Hz. This frequency can easily be related, for example, to the rotational speed of a particular motor. Thus the acceleration spectrum shows a strong evidence that the motor might be the cause of vibration. If the motor is causing the excessive vibrations, changing either the motor or its speed of operation might avoid resonance and hence the problem of excessive vibrations.



- Spectrum or frequency analyzers can be used for signal analysis. These devices analyze a signal in the frequency domain by separating the energy of the signal into various frequency bands. The separation of signal energy into frequency bands is accomplished through a set of filters. The analyzers are usually classified according to the type of filter employed. For example, if an octave band filter is used, the spectrum analyzer is called an *octave band analyzer*.
- In recent years, digital analyzers have become quite popular for real-time signal analysis. In a real-time frequency analysis, the signal is continuously analyzed over all the frequency bands. Thus the calculation process must not take more time than the time taken to collect the signal data. Real-time analyzers are especially useful for machinery health monitoring, since a change in the noise or vibration spectrum can be observed at the same time that change in the machine occurs.
- There are two types of real-time analysis procedures: the digital filtering method and the fast Fourier transform (FFT) method. The digital filtering method is best suited for constant-percent bandwidth analysis, the FFT method for constant-bandwidth analysis.
- A bandpass filter is a circuit that permits the passage of frequency components of a signal over a frequency band and rejects all other frequency components of the signal. A filter can be built by using, for example, resistors, inductors, and capacitors.

Dynamic Testing of Machines and Structures

- The dynamic testing of machines and structures involves finding their deformation at a critical frequency. This can be done using the following two approaches.
- Using operational deflection-shape measurements. In this method, the forced dynamic deflection shape is measured under the steady-state (operating) frequency of the system.
- For the measurement, an accelerometer is mounted at some point on the machine (structure) as a reference, and another moving accelerometer is placed at several other points, and in different directions, if necessary. Then the magnitudes and the phase differences between the moving and reference accelerometers at all the points under steady-state operation of the system are measured. By plotting these measurements, we can find how the various parts of the machine (structure) move relative to one another and also absolutely.

- The deflection shape measured is valid only for the forces/frequency associated with the operating conditions; as such, we cannot get information about deflections under other forces and/or frequencies. However, the measured deflection shape can be quite useful. For example, if a particular part or location is found to have excessive deflection, we can stiffen that part or location. This, in effect, increases the natural frequency beyond the operational frequency range of the system.
- Using modal testing. Since any dynamic response of a machine (structure) can be obtained as a combination of its modes, a knowledge of the mode shapes, modal frequencies, and modal damping ratios constitutes a complete dynamic description of the machine (structure).

Experimental Modal Analysis

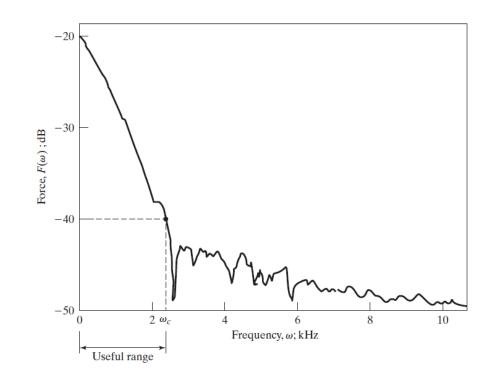
- *Experimental modal analysis*, also known as *modal analysis* or *modal testing*, deals with the determination of natural frequencies, damping ratios, and mode shapes through vibration testing. Two basic ideas are involved:
- 1. When a structure, machine, or any system is excited, its response exhibits a sharp peak at resonance when the forcing frequency is equal to its natural frequency when damping is not large.
- 2. The phase of the response changes by 180° as the forcing frequency crosses the natural frequency of the structure or machine, and the phase will be 90° at resonance.

- The measurement of vibration requires the following hardware:
- 1. An **exciter** or source of vibration to apply a known input force to the structure or machine.
- 2. A **transducer** to convert the physical motion of the structure or machine into an electrical signal.
- 3. A **signal conditioning amplifier** to make the transducer characteristics compatible with the input electronics of the digital data acquisition system.
- 4. An **analyzer** to perform the tasks of signal processing and modal analysis using suitable software.

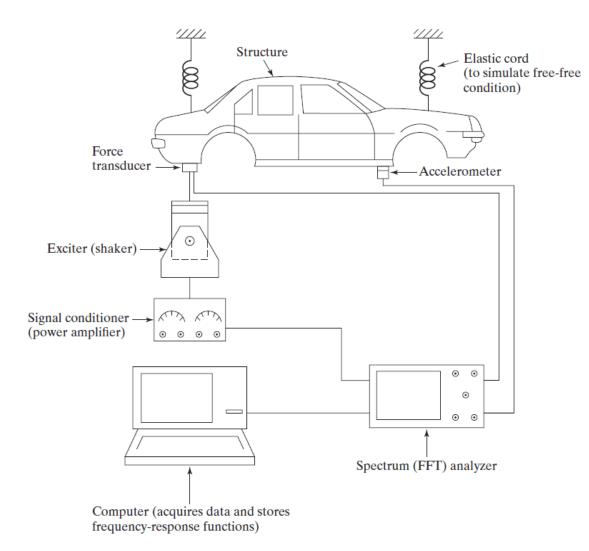
- Exciter. The exciter may be an electromagnetic shaker or an impact hammer.
- The electromagnetic shaker can provide large input forces so that the response can be measured easily. Also the output of the shaker can be controlled easily if it is of electromagnetic type. The excitation signal is usually of a swept sinusoidal or a random type signal. In the swept sinusoidal input, a harmonic force of magnitude *F* is applied at a number of discrete frequencies over a specific frequency range of interest. At each discrete frequency, the structure or machine is made to reach a steady state before the magnitude and phase of the response are measured. If the shaker is attached to the structure or machine being tested, the mass of the shaker will influence the measured response (known as the *mass loading effect*).
- As such, care is to be taken to minimize the effect of the mass of the shaker. Usually the shaker is attached to the structure or machine through a short thin rod, called a *stringer*, to isolate the shaker, reduce the added mass, and apply the force to the structure or machine along the axial direction of the stringer. This permits the control of the direction of the force applied to the structure or machine.

- The impact hammer is a hammer with a built-in force transducer in its head. The impact hammer can be used to hit or impact the structure or machine being tested to excite a wide range of frequencies without causing the problem of mass loading. The impact force caused by the impact hammer, which is nearly proportional to the mass of the hammer head and the impact velocity, can be found from the force transducer embedded in the head of the hammer. The response of the structure or machine to an impulse is composed of excitations at each of the natural frequencies of the structure or machine.
- Although the impact hammer is simple, portable, inexpensive, and much faster to use than a shaker, it is often not capable of imparting sufficient energy to obtain adequate response signals in the frequency range of interest. It is also difficult to control the direction of the applied force with an impact hammer.

- A typical frequency response of a structure or machine obtained using an impact hammer is shown. The shape of the frequency response is dependent on the mass and stiffness of both the hammer and the structure or machine.
- Usually, the useful range of frequency excitation is limited by a cutoff frequency, ω_c , which implies that the structure or machine did not receive sufficient energy to excite modes beyond ω_c . The value of ω_c is often taken as the frequency where the amplitude of the frequency response reduces by 10 dB to 20 dB from its maximum value.



- **Transducer.** Among the transducers, the piezoelectric transducers are most popular. A piezoelectric transducer can be designed to produce signals proportional to either force or acceleration.
- In an accelerometer, the piezoelectric material acts as a stiff spring that causes the transducer to have a resonant or natural frequency. Usually, the maximum measurable frequency of an accelerometer is a fraction of its natural frequency. Strain gages can also be used to measure the vibration response of a structure or machine.
- **Signal Conditioner.** Since the output impedance of transducers is not suitable for direct input into the signal analysis equipment, signal conditioners, in the form of charge or voltage amplifiers, are used to match and amplify the signals before signal analysis.
- Analyzer. The response signal, after conditioning, is sent to an analyzer for signal processing. A type that is commonly used is the *fast Fourier transform (FFT) analyzer*.
- Such an analyzer receives analog voltage signals (representing displacement, velocity, acceleration, strain, or force) from a signal-conditioning amplifier, filter, and digitizer for computations. It computes the discrete frequency spectra of individual signals as well as cross-spectra between the input and the different output signals. The analyzed signals can be used to find the natural frequencies, damping ratios, and mode shapes in either numerical or graphical form.



- The general arrangement for the experimental modal analysis of a structural or mechanical system is shown.
- Note that all the equipment is to be calibrated before it is used. For example, an impact hammer is used more frequently in experimental stress analysis. The reason is that it is more convenient and faster to use than a shaker.
- An impact hammer consists of a force transducer or load cell built into the head (or tip) of the hammer. The built-in force transducer is to be calibrated dynamically whenever the head or tip is changed.
- Similarly, the transducers, along with the signal conditioners, are to be calibrated with respect to magnitude and phase over the frequency range of interest.

Test yourself

- Indicate whether each of the following statements is true or false:
- 1) A strain gage is a variable-resistance transducer.
- 2) The value of the gage factor of a strain gage is given by the manufacturer.
- 3) The voltage output of an electromagnetic transducer is proportional to the relative velocity of the coil.
- 4) The principle of the electrodynamic transducer can be used in vibration exciters.
- 5) A seismometer is also known as a vibrometer.
- 6) All vibration-measuring instruments exhibit phase lag.

- Indicate whether each of the following statements is true or false:
- 7) The time lag is important when measuring harmonic motion of frequency ω .
- 8) The Scotch yoke mechanism can be used as a mechanical shaker.
- 9) The time response of a system gives better information on energy distribution than does the frequency response.
- 10) A spectrum analyzer is a device that analyzes a signal in the frequency domain.
- 11) The complete dynamic response of a machine can be determined through modal testing.
- 12) The spectrum analyzers are also known as fast Fourier transform (FFT) analyzers.

and of the Lecture

